

Measuring the Neutron-Star EOS with Gravitational Waves from Binary Inspiral

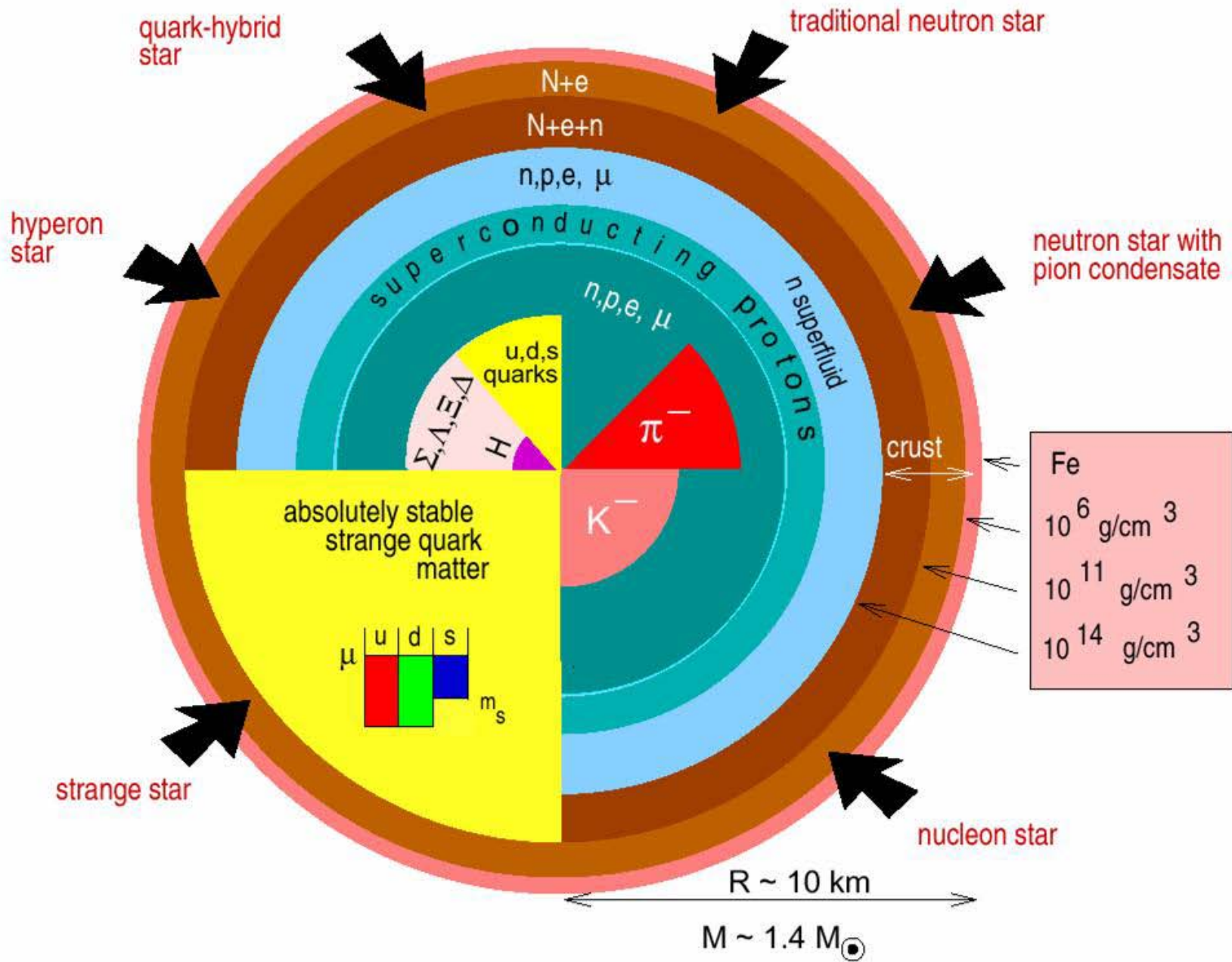
I. Astrophysical Constraints

Jocelyn Read, Ben Lackey, Ben Owen, JF

II. NRDA NS-NS

Jocelyn Read, Charalampos Markakis,
Masaru Shibata, Koji Uryu, Jolien
Creighton, JF; Hinderer, Read, Lackey

BH-NS Shibata, Keisuke Taniguchi



Systematizing Constraints on EOS

Want parameterized EOS $p=p(\rho)$

p = pressure

ρ = rest-mass density

ϵ = energy density

- Number of parameters
< number of astrophysical observables
- Number of parameters large enough to accurately specify an EOS. Test by accuracy in matching the universe of candidate EOSs.

Systematizing Constraints on EOS

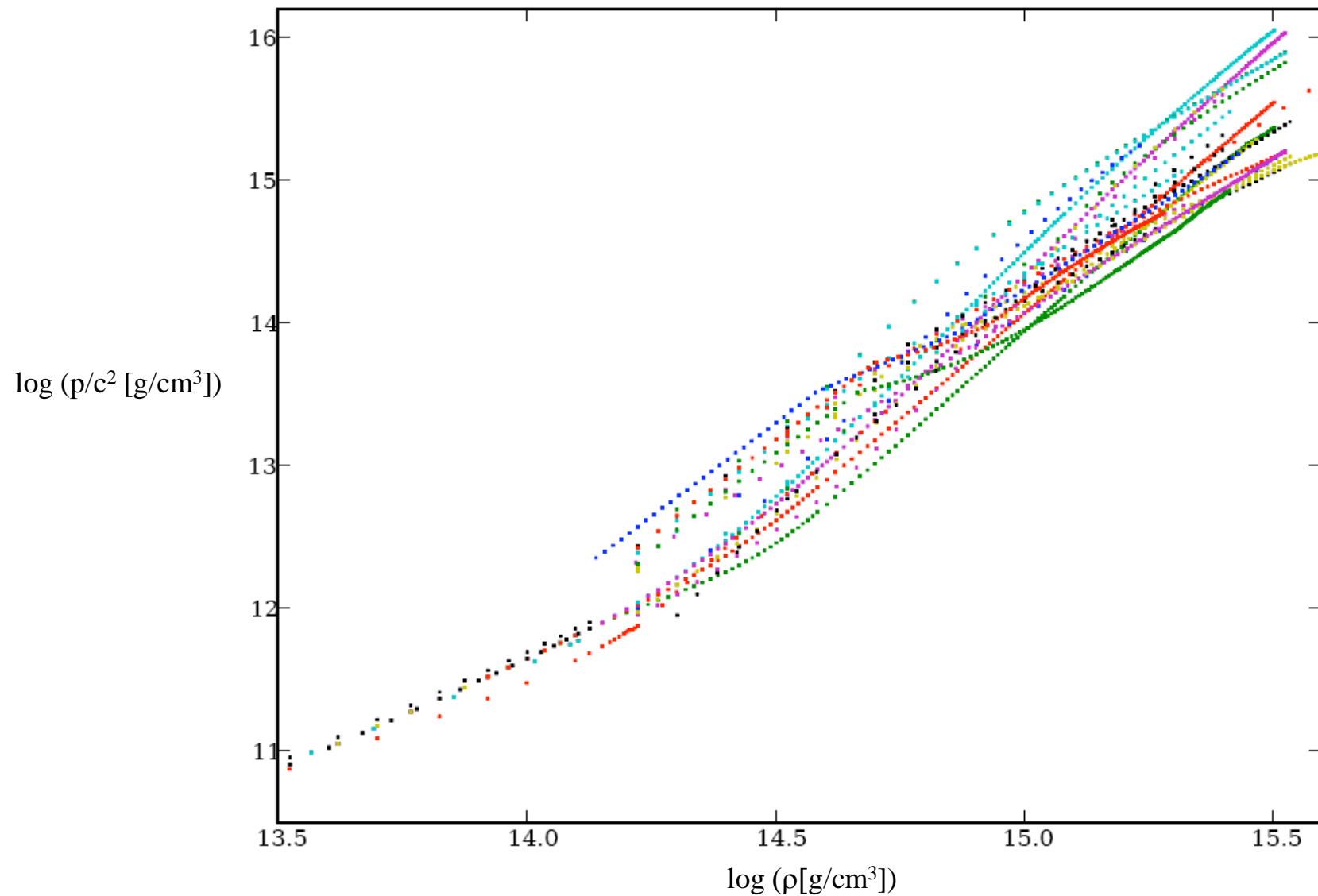
Difficulty:

No small, model-independent set of fundamental parameters.

Systematizing Constraints on EOS

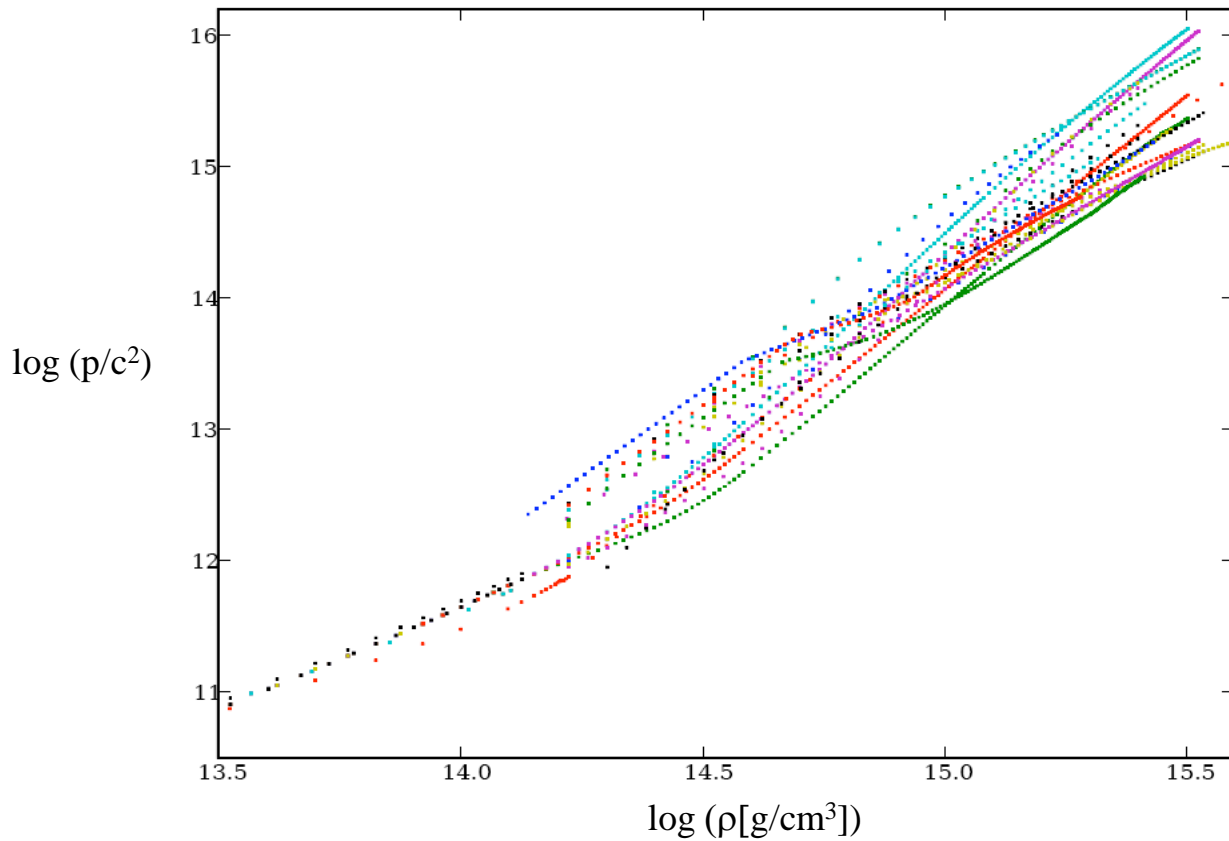
Instead, try phenomenological parametrization.

Candidate EOSs



$\log p [\log(\rho)]$ roughly piecewise linear

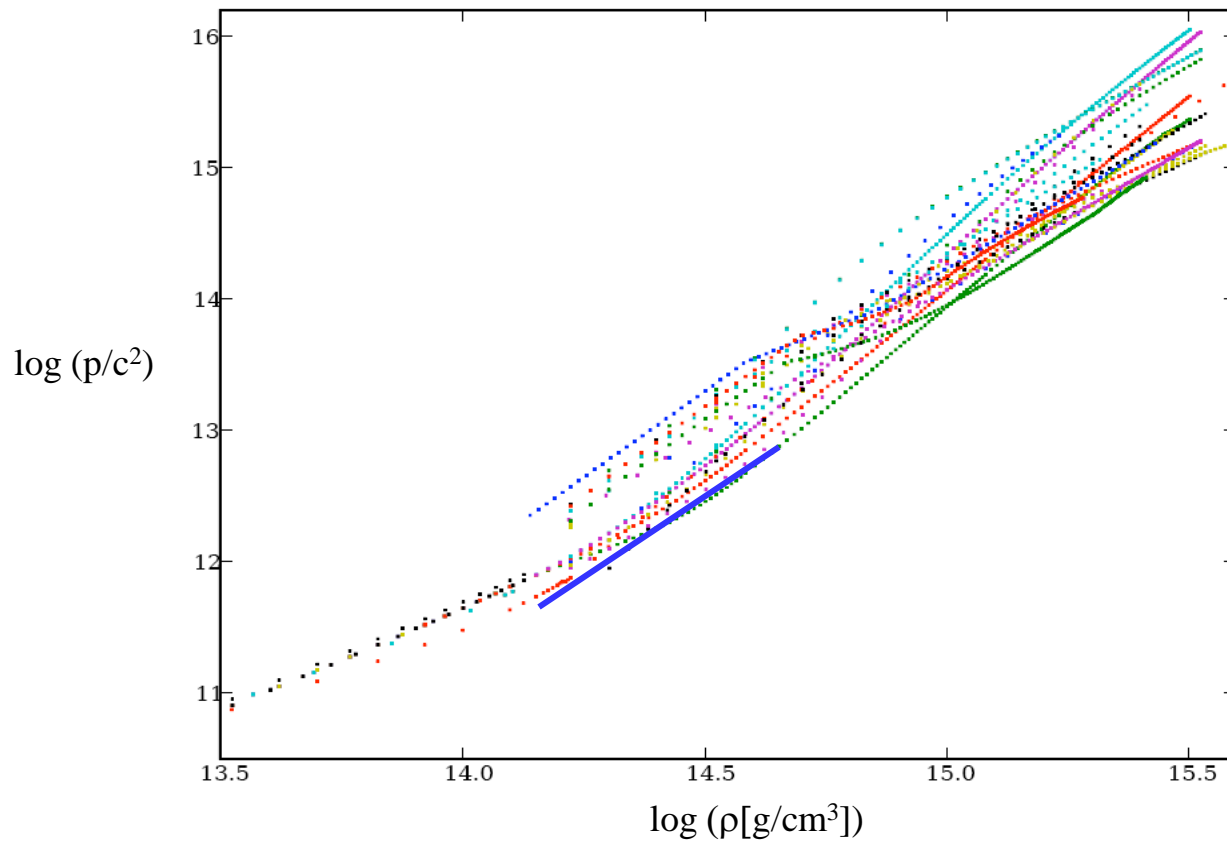
Candidate EOSs



Specify $\Gamma = \frac{d \log p}{d \log \rho}$ at 3 different densities

Translate $p(\rho)$ curve up or down

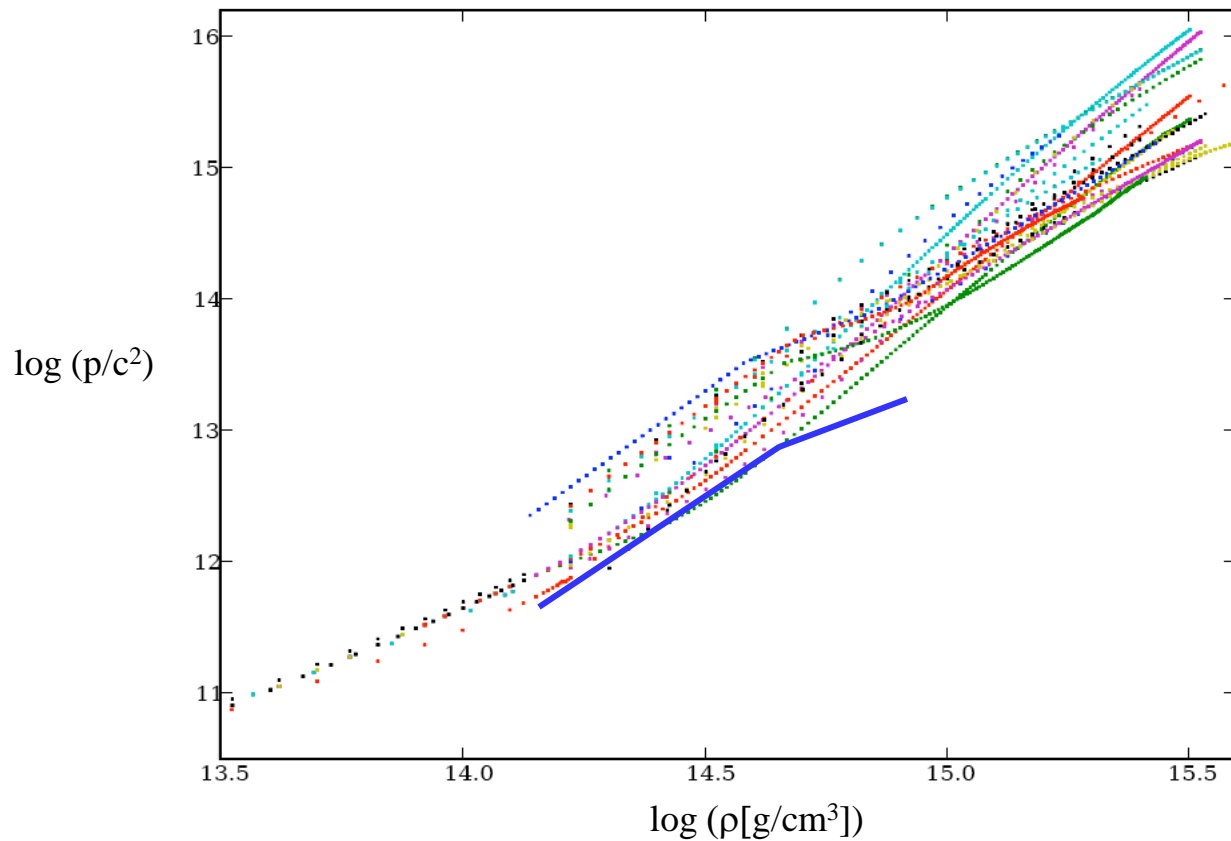
Candidate EOSs



Specify $\frac{d \log p}{d \log \rho}$ at 3 different densities

Translate $p(\rho)$ curve up or down

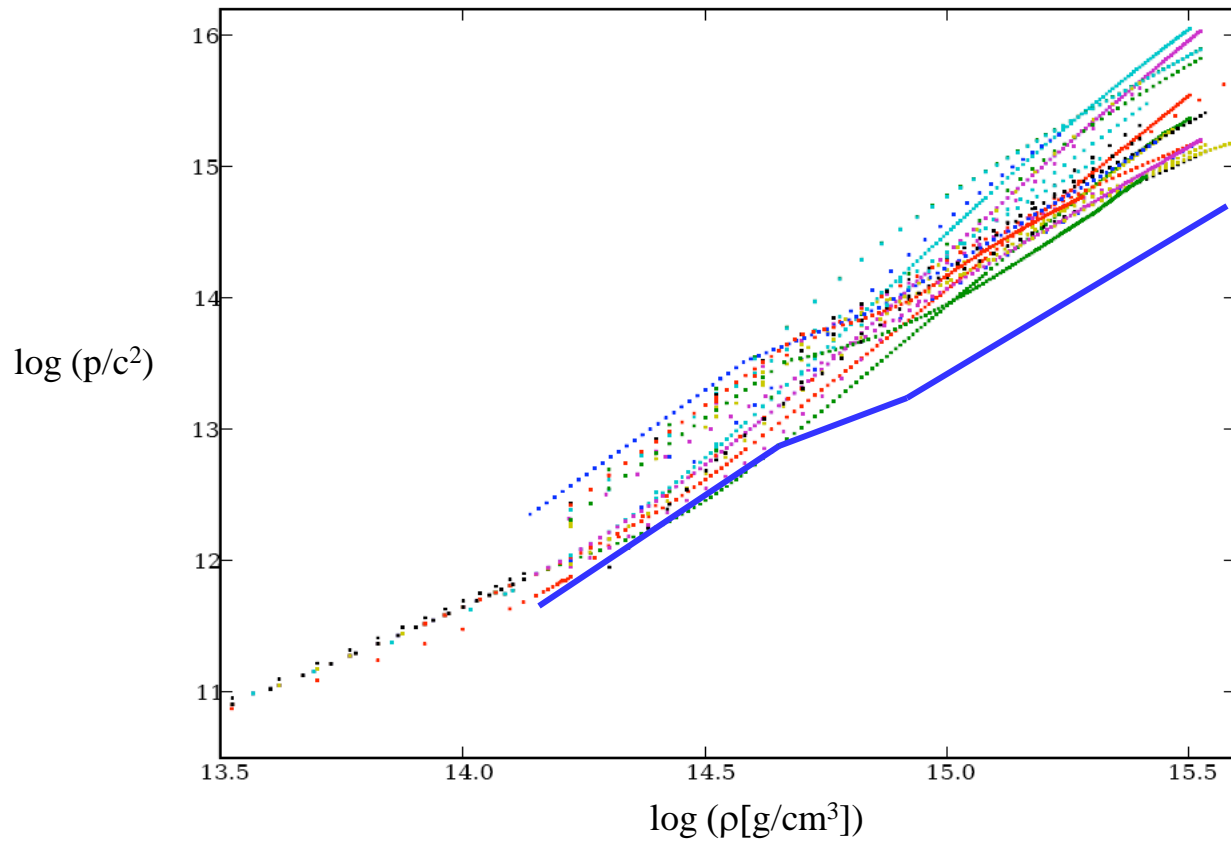
Candidate EOSs



Specify $\frac{d \log p}{d \log \rho}$ at 3 different densities

Translate $p(\rho)$ curve up or down

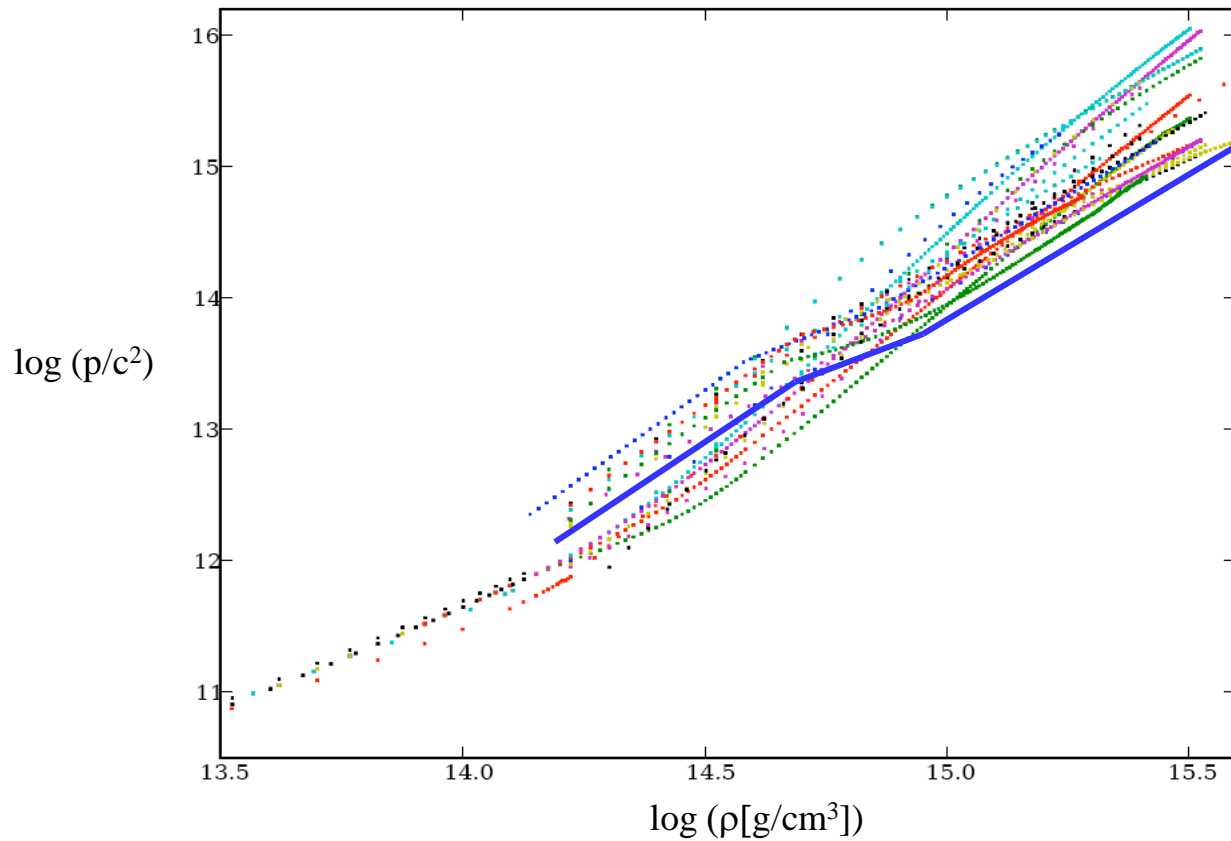
Candidate EOSs



Specify $\frac{d \log p}{d \log \rho}$ at 3 different densities

Translate $p(\rho)$ curve up or down

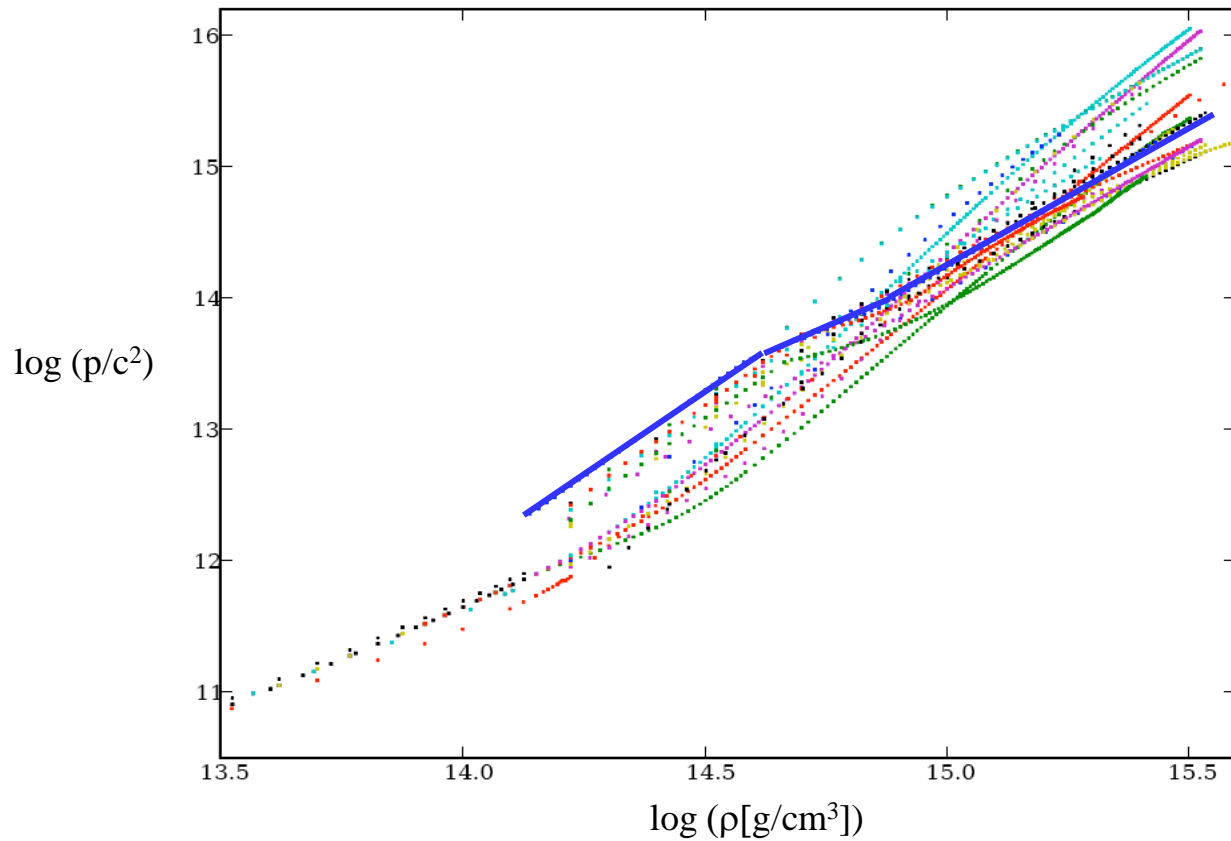
Candidate EOSs



Specify $\frac{d \log p}{d \log \rho}$ at 3 different densities

Translate $p(\rho)$ curve up or down

Candidate EOSs

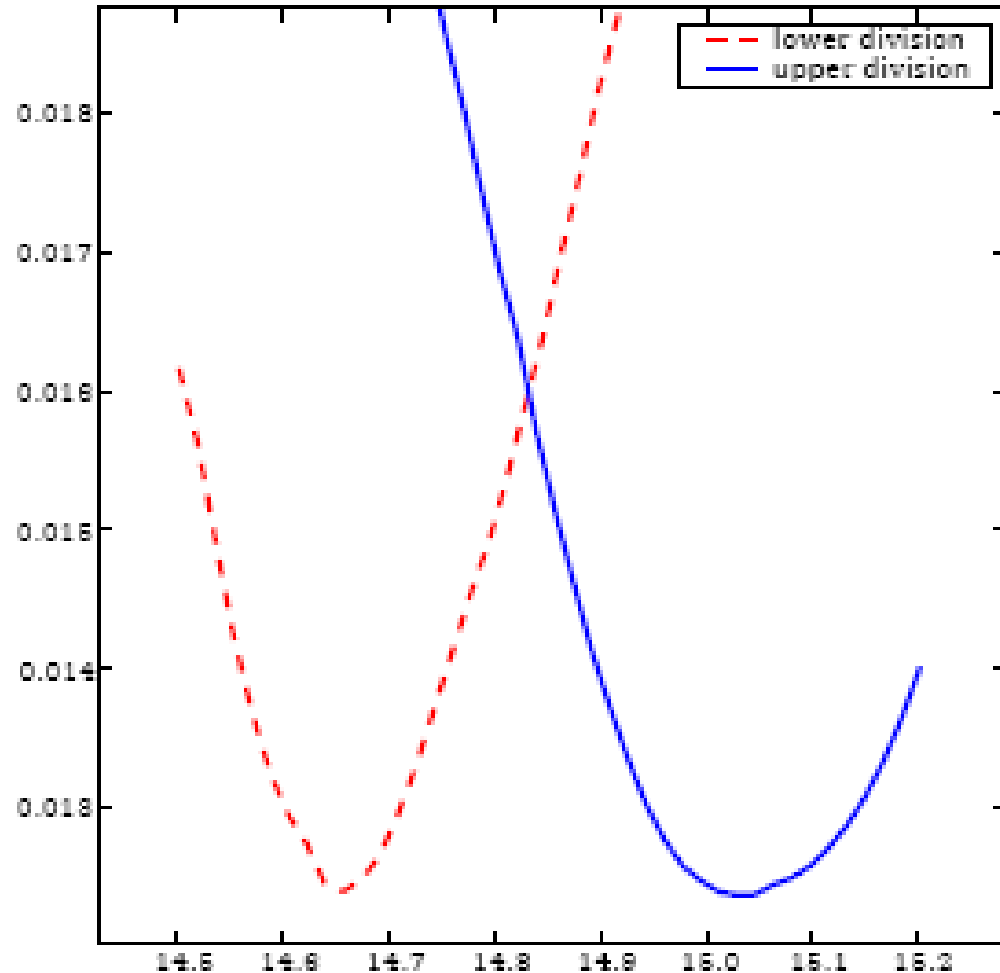


Specify $\Gamma = \frac{d \log p}{d \log \rho}$ at 3 different densities

Translate $p(\rho)$ curve up or down

Happily, universe of candidates has preferred dividing densities:

Average rms error in match to universe of candidate EOSs



$\log \rho_{\text{dividing}} [\text{g/cm}^3]$

These are piecewise polytropes

The parameters: p_* , Γ_1 , Γ_2 , Γ_3

$$p_* = p(2.5 \times 10^{14} \text{ g/cm}^3)$$

$$p(\rho) = \begin{cases} \text{Known} & \rho < \rho_{\text{nuclear}} \equiv \rho_1 \\ K_1 \rho^{\Gamma_1} & \rho_1 < \rho < \rho_2 \\ K_2 \rho^{\Gamma_2} & \rho_2 < \rho < \rho_3 \\ K_m \rho^{\Gamma_3} & \rho > \rho_3 \end{cases}$$

K_i fixed by continuity (Ipser, Vuille '99)

Hard theoretical constraints:

- EOS known below nuclear density
- Causality: $v_{\text{sound}} < 1 \Rightarrow \frac{dp}{d\epsilon} < 1$

Hard astrophysical constraints:

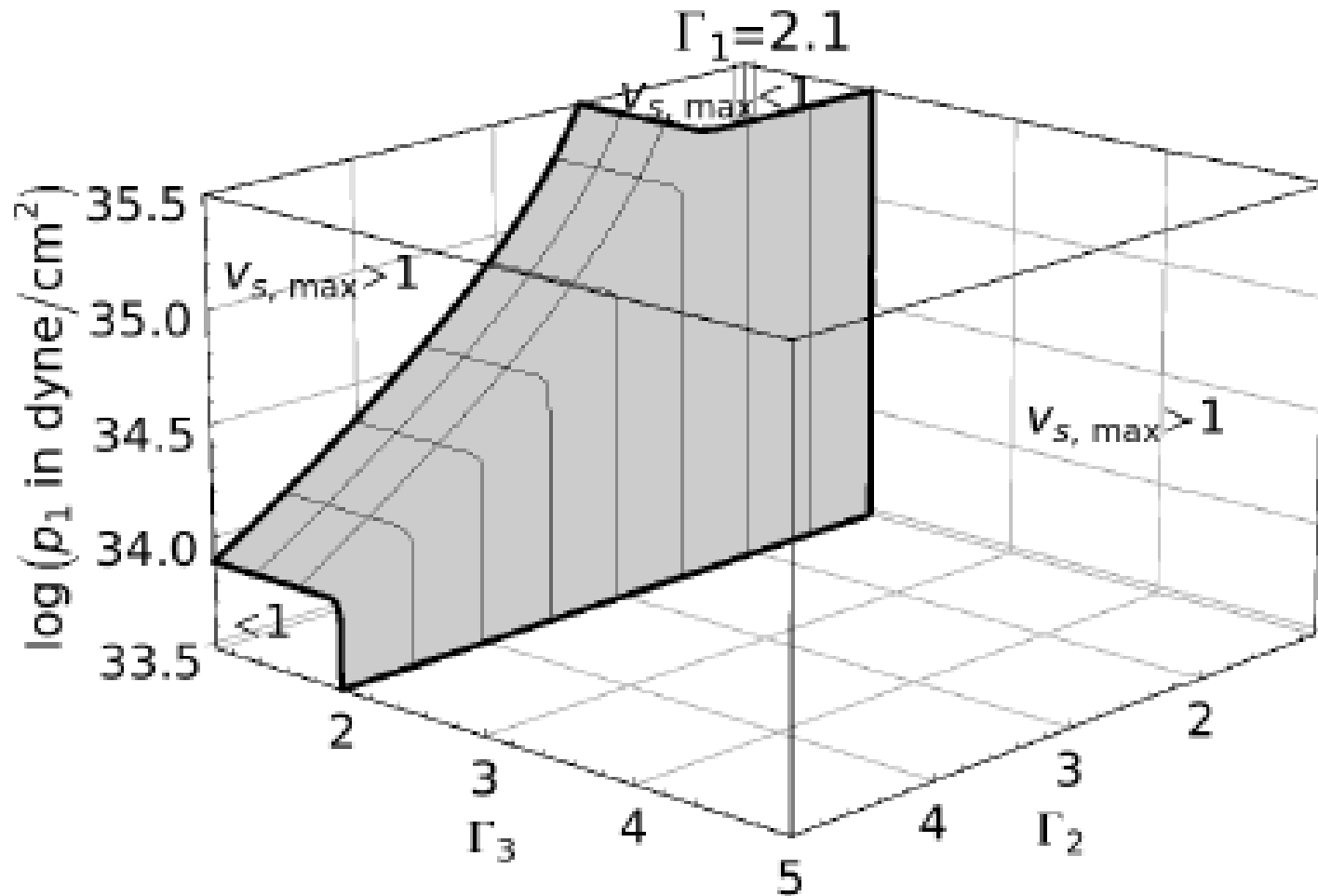
$$M_{\max} > 1.7 M_{\odot} \quad (95\% \text{ confidence J075-1907})$$

$$\Omega_{\max} > 716 \text{ Hz} \quad (\text{PSR J1748-2446})$$

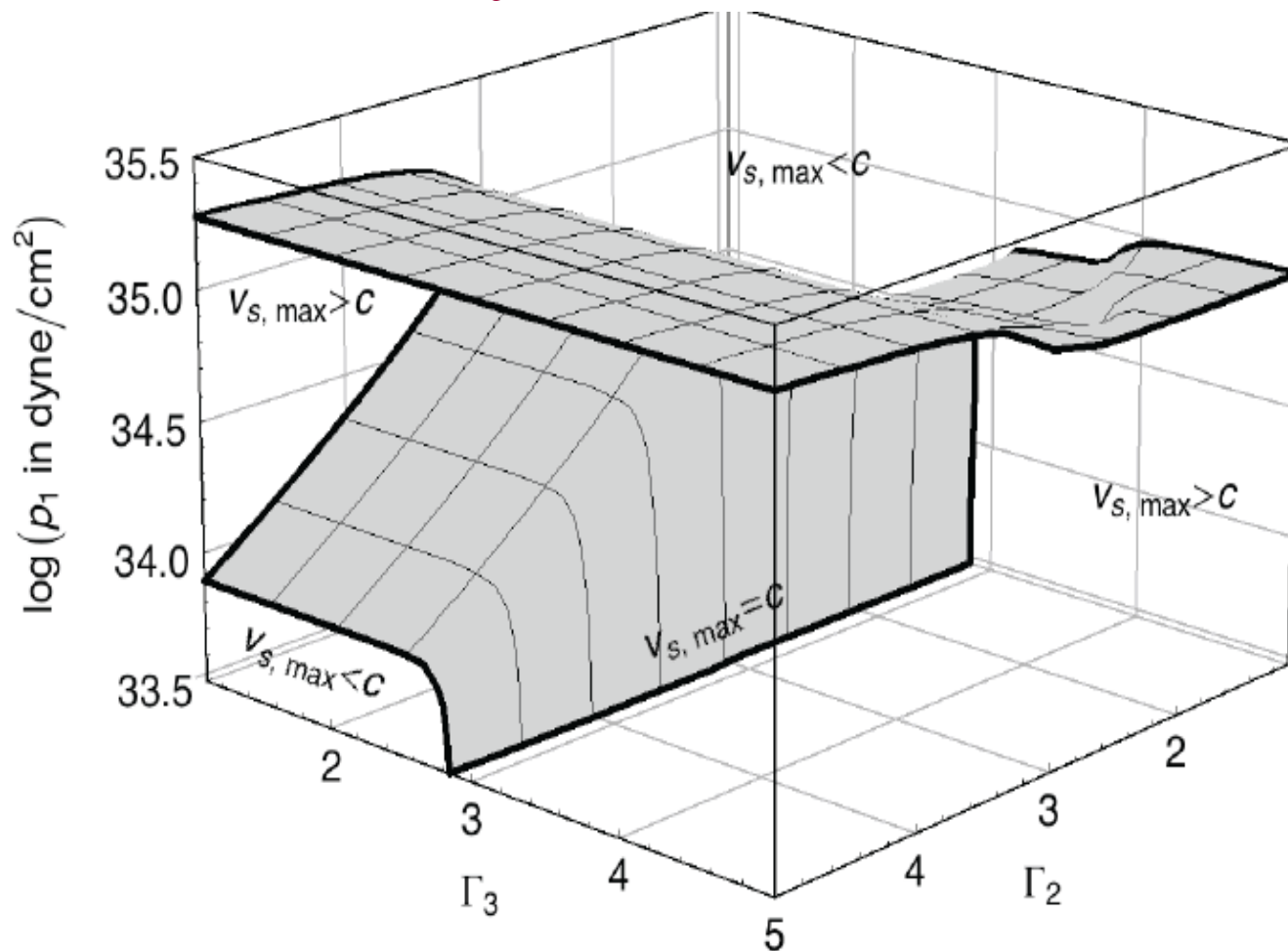
Of these, only the first –
the largest observed mass – significantly
constrains the EOS parameter space

It only constrains the parameters in
medium- to high-density part of core EOS.

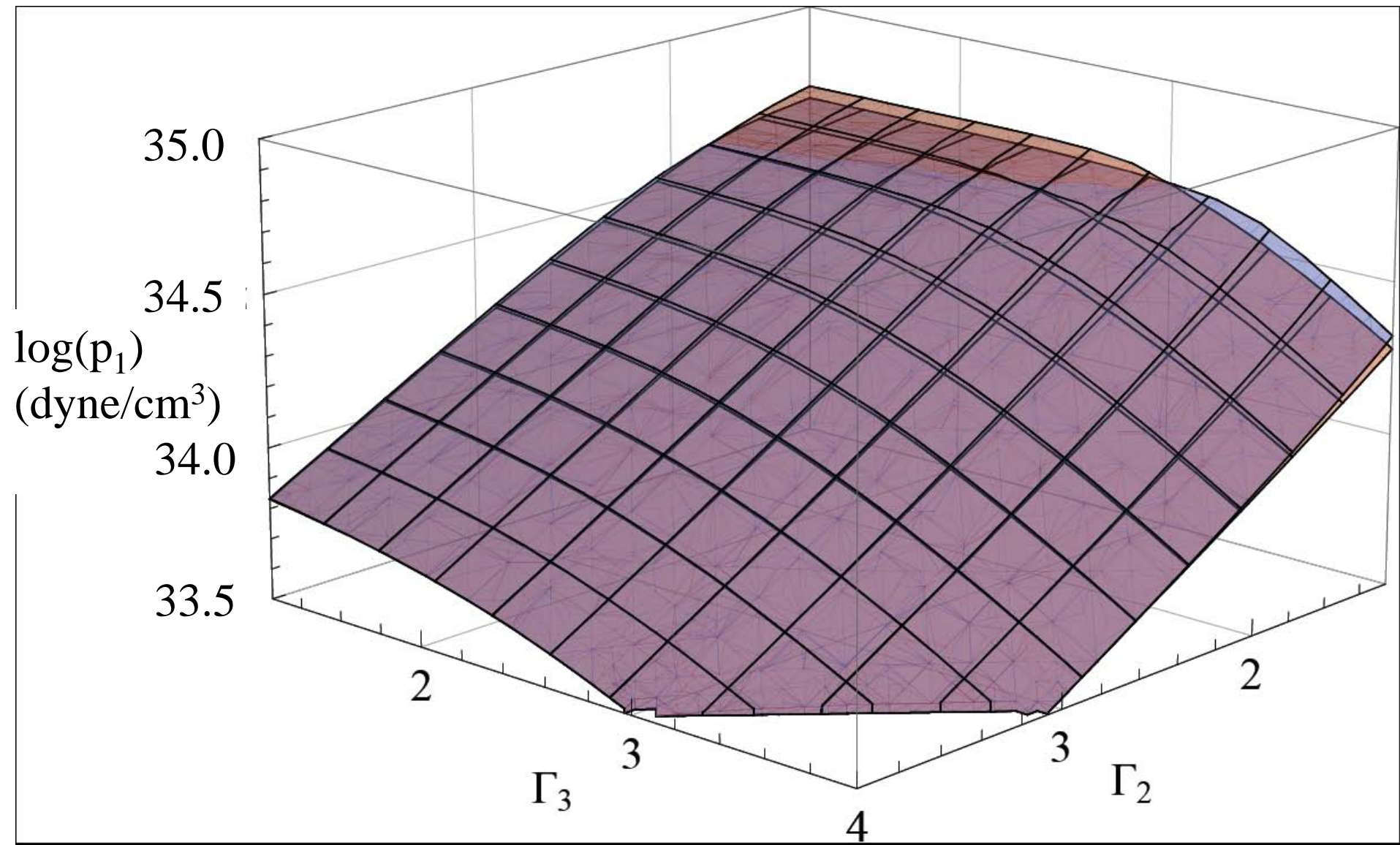
Causality, $v_{\text{sound}} < c$, removes the region above and in front of the surface, where the EOS is too stiff



The constraint is weaker when one demands causality only for densities below the largest density occurring in stars based on the EOS: the central density of the maximum mass star

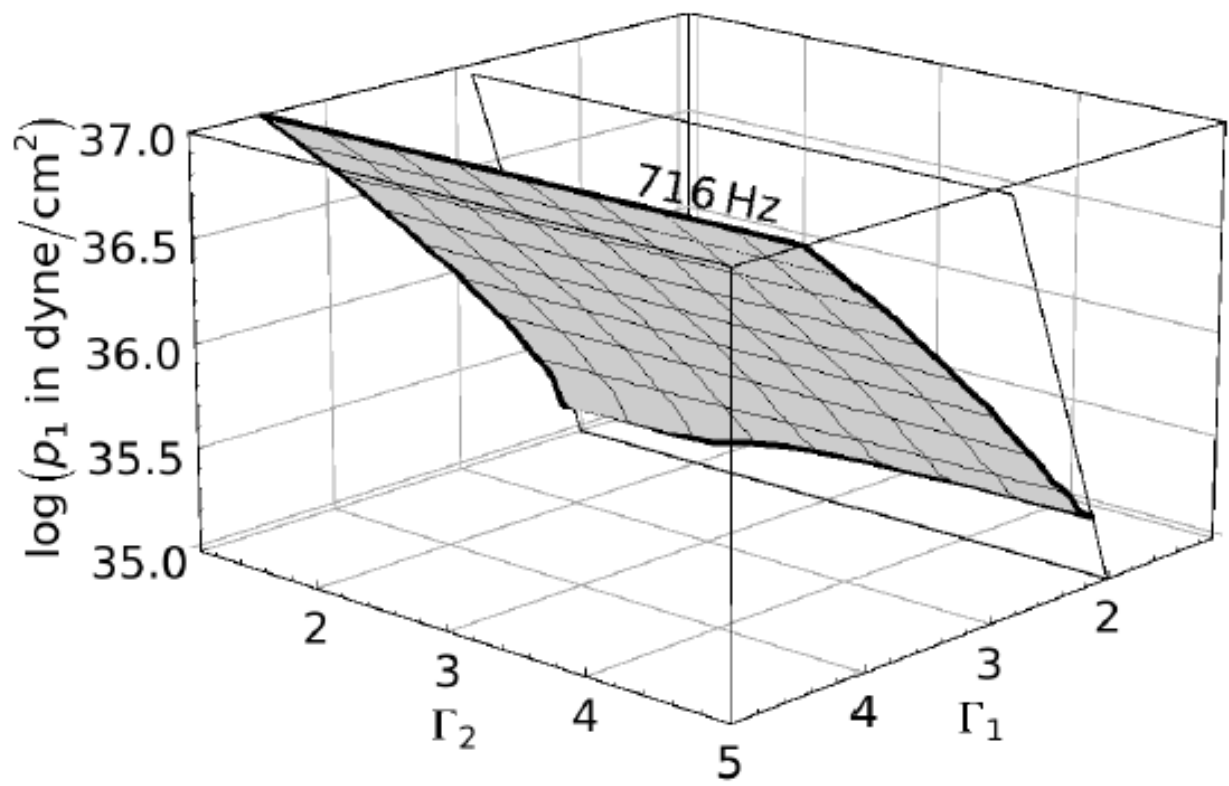


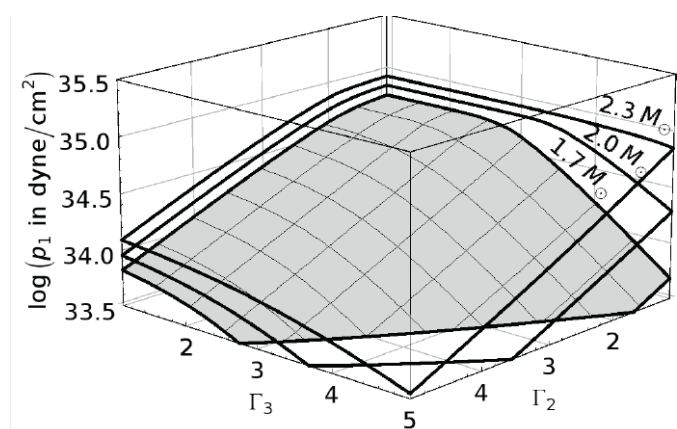
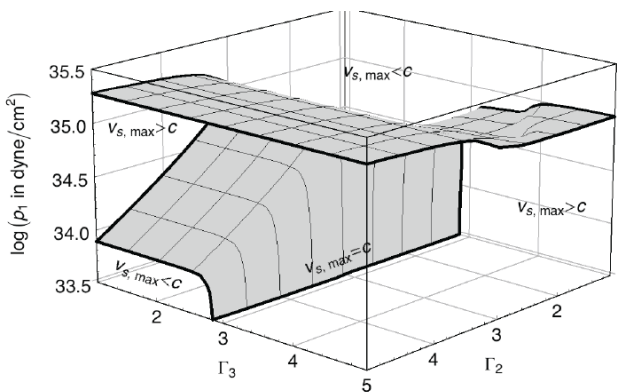
$M_{\text{max}} > 1.7 M_{\odot}$ rules out region below surface
(where EOS too soft to prevent collapse)



Largest confirmed spin, 716 Hz, rules out only EOSs already regarded as unphysically stiff.

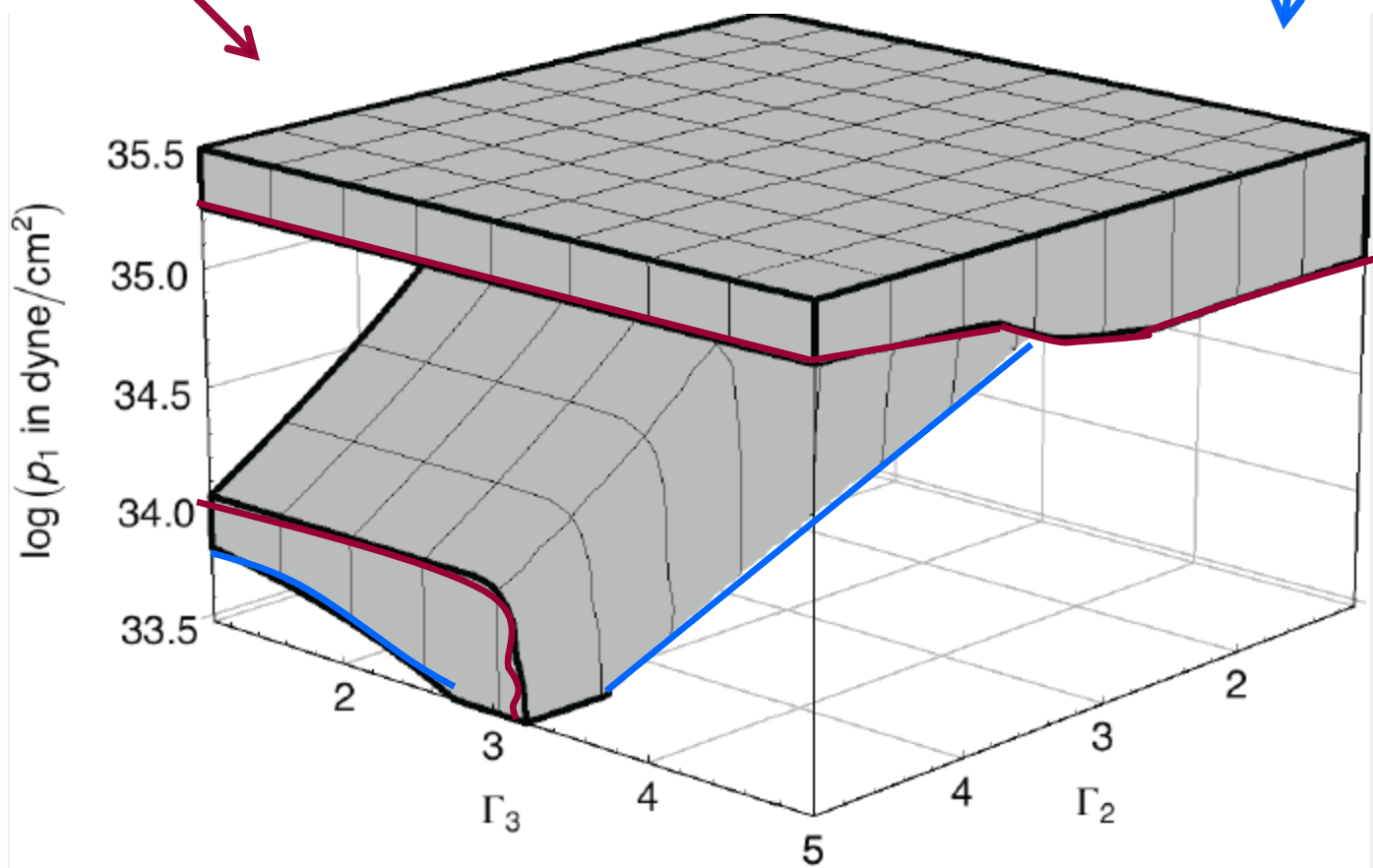
Because the **causality**, **maximum-mass** and **maximum-spin** constraints are inequalities, they only restrict the EOS to a region on one side of a surface in parameter space.





$v_{\text{sound}} < c$

$M > 1.7 M_\odot$



Measuring two properties of a single star restricts the EOS to a single surface (thickened by the error bars of the measurement).

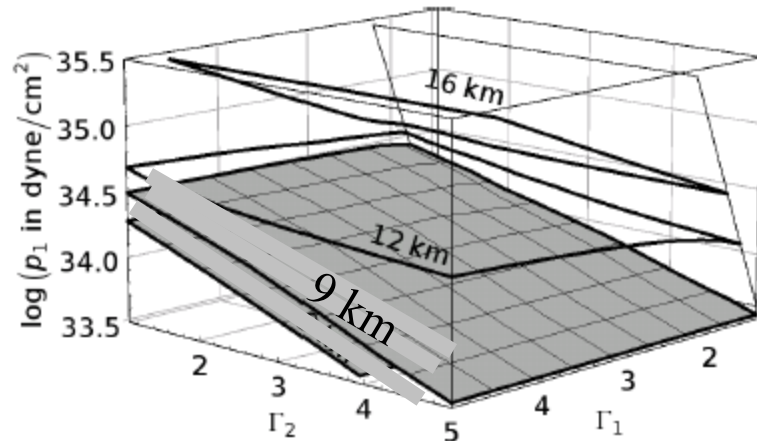
Example:

Observing mass and radius restricts EOS to the surface

$$R(p_*, \Gamma_1, \Gamma_2, M_{\text{observed}}) = R_{\text{observed}}$$

These constraints are model dependent, but some have small estimated systematic errors.
(Ozel and collaborators, Ozel&Psaltis)

e.g., $M = 1.4M_{\odot}$, $R = 9 \text{ km}$



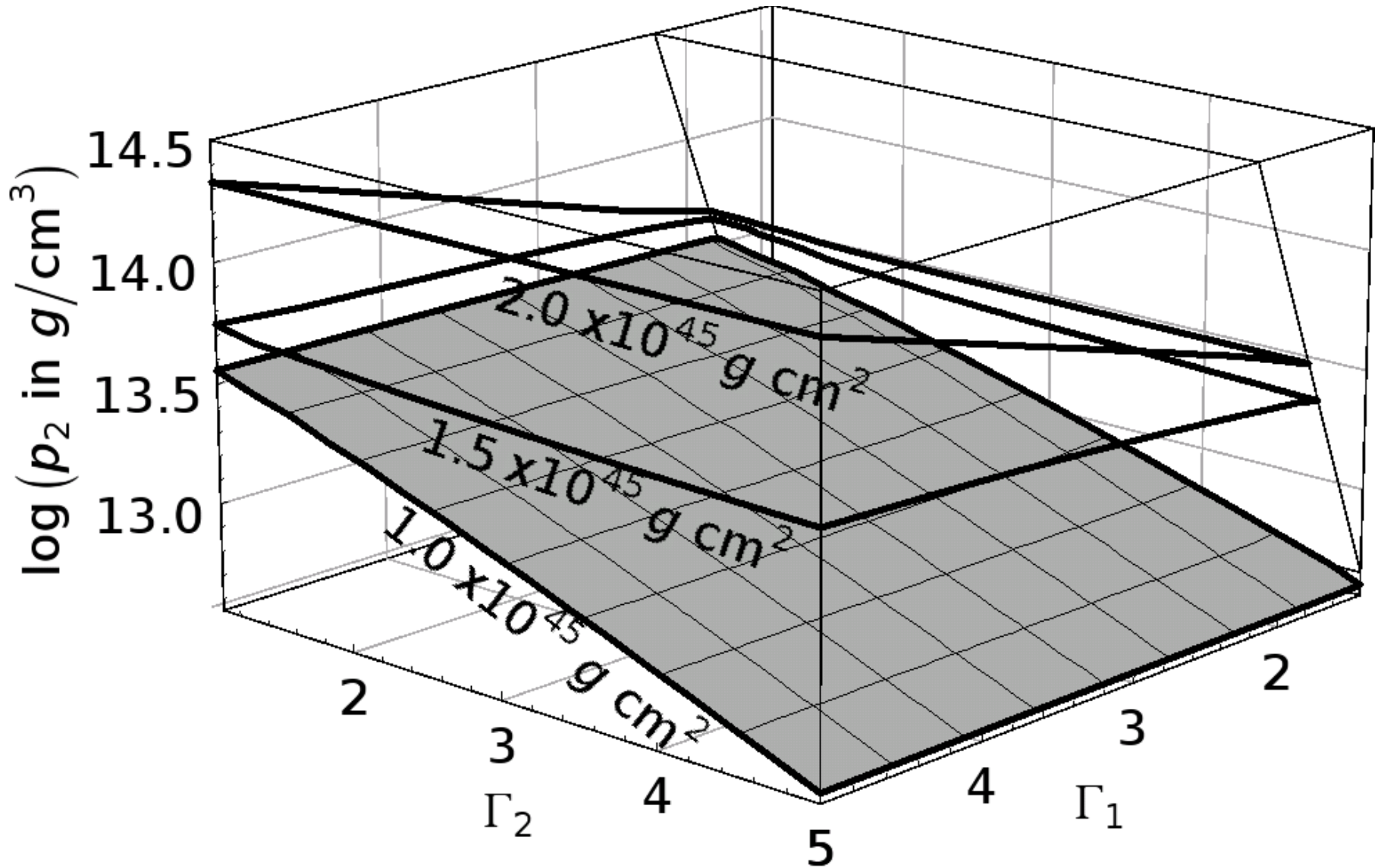
Future Constraints

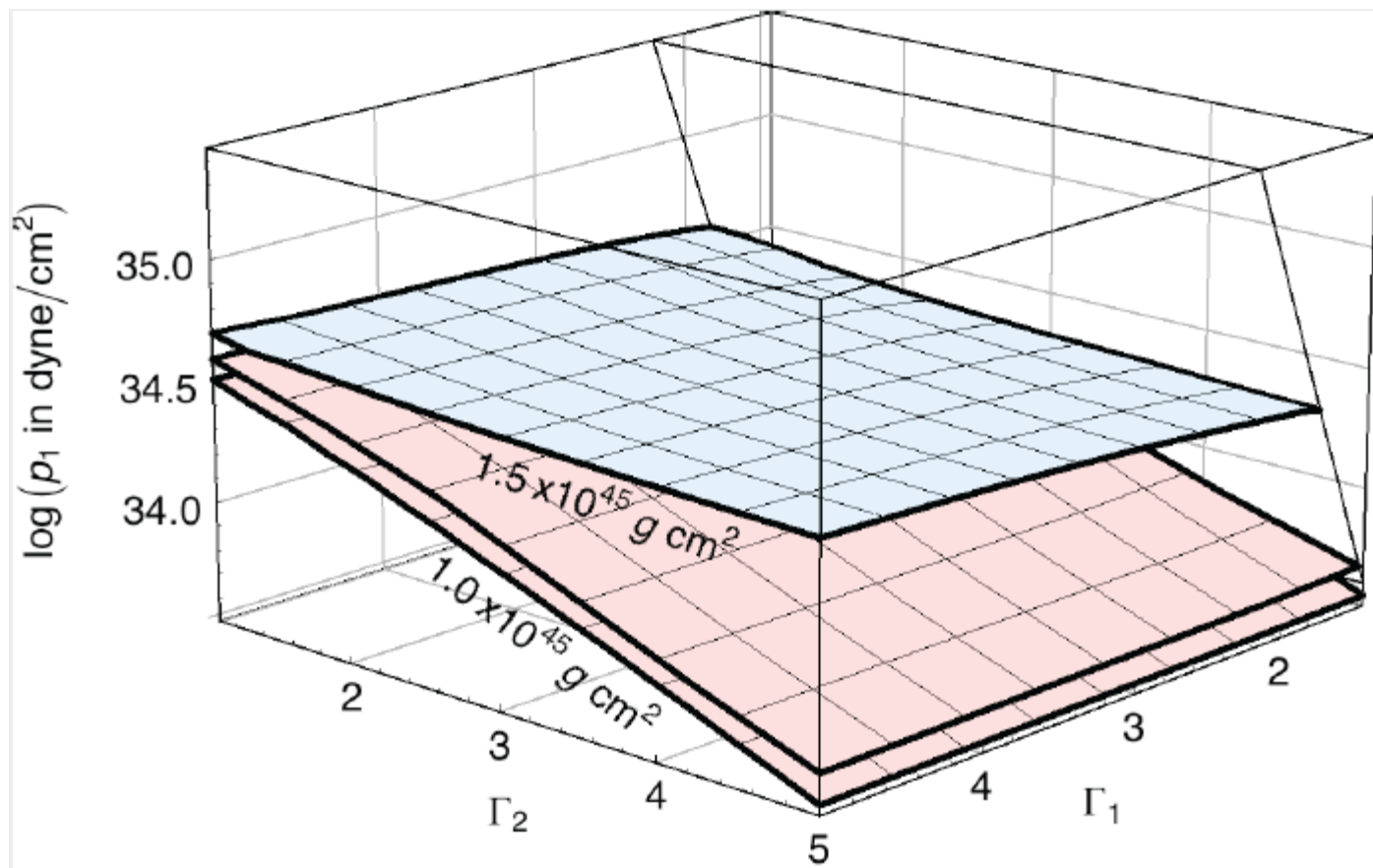
I and M for the same star, PSR J0737-3039A
in fastest binary pulsar system

M and measure of departure of NS-NS
waveform from point-particle inspiral

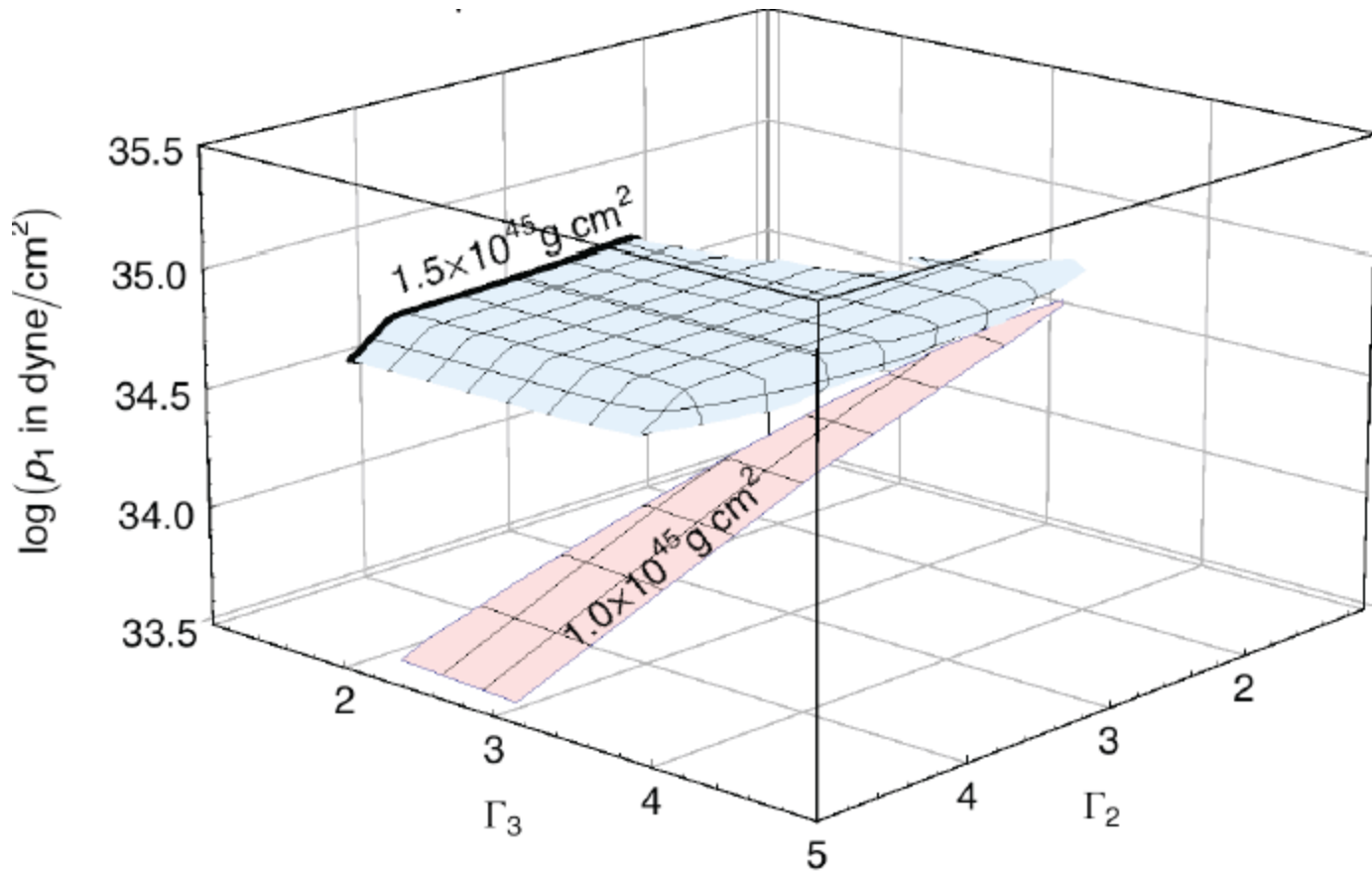
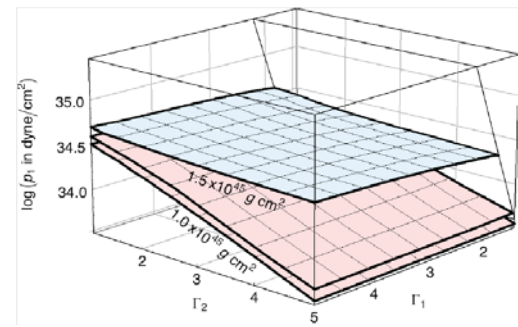
σ_{mode} : Mode frequencies for merged star
prior to becoming black hole.
Simultaneously know M and inspiral
waveform, but EOS hot, and
theoretical σ model dependent

Future observation of moment of inertia to 10% from double pulsar PSR J0737-3039A?

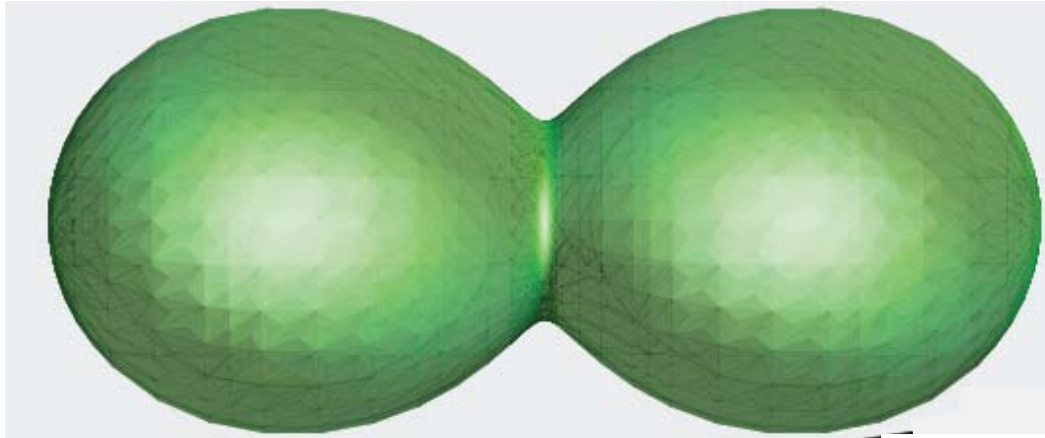




Constraint surfaces for two values of I for a $1.4 M_{\odot}$ star, combined with the causality and maximum-mass constraints.



Departure from point-particle inspiral at lower frequency for NS with larger radius, because the tidal force is greater.



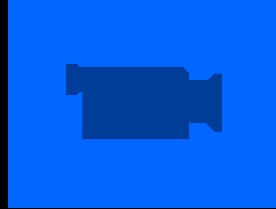
Stiffer EOS

$$p(1.7\rho_{\text{nuclear}}) = 10^{34.9} \text{ dyne/cm}^2$$

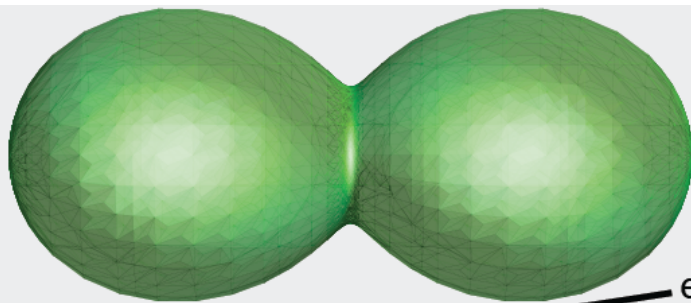


Softer EOS

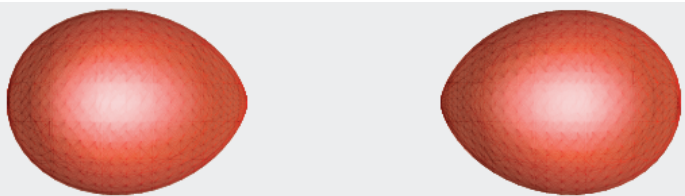
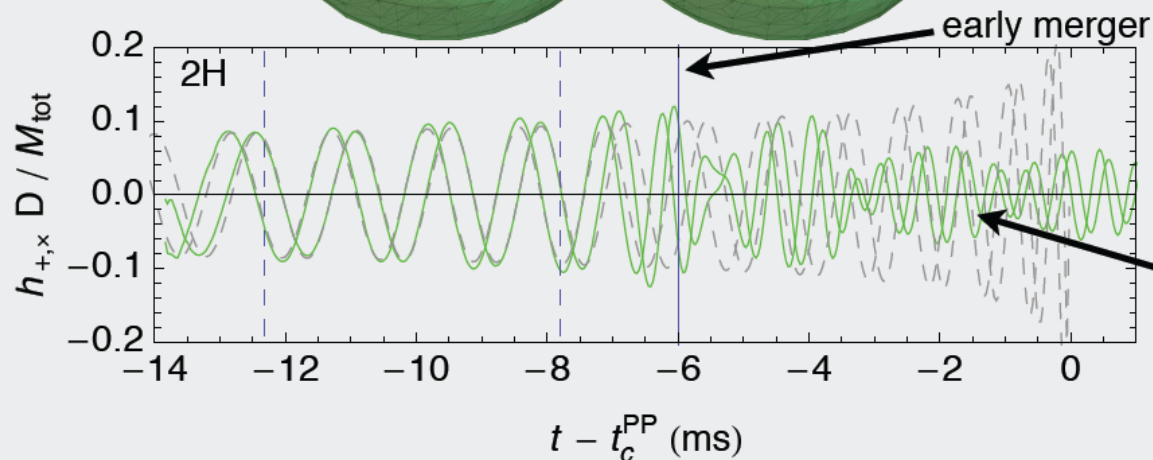
$$p(1.7\rho_{\text{nuclear}}) = 10^{34.1} \text{ dyne/cm}^2$$



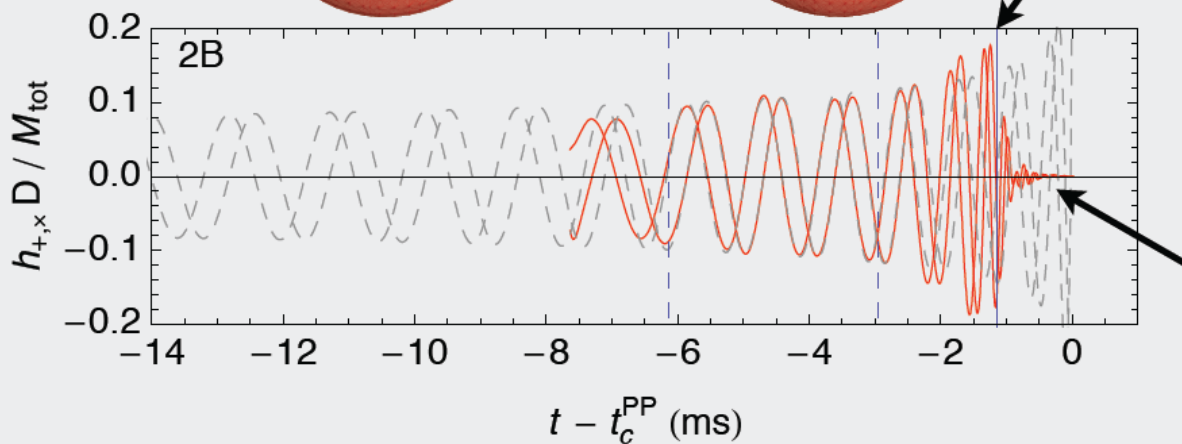
Simulations by Markakis, Shibata, Uryu –
a first cut, using a few orbits before merger.



Model 2H (stiff)
 $\rho_1 = 10^{34.9}$ dyne cm^{-3}
 $\Gamma_1 = \Gamma_2 = \Gamma_3 = 3$
Isolated radius: 15.2 km



Model 2B (soft)
 $\rho_1 = 10^{34.1}$ dyne cm^{-3}
 $\Gamma_1 = \Gamma_2 = \Gamma_3 = 3$
Isolated radius: 9.7 km



Estimated accuracy with which radius
 can be extracted from inspiral waveform
 for two $1.4M_{\odot}$ stars at 100 Mpc

R (km)	Broadband	Narrow band 1150 Hz
R=10.3	$\pm .61$	$\pm .57$
R=11.25	$\pm .78$	$\pm .78$
R=13.4	± 1.75	± 2.13

Criterion: $\sqrt{\langle h_a - h_b | h_a - h_b \rangle} > 1$

And a larger R requires a stiffer EOS –
larger pressure p_* at average density of star.

$\log p_*$ (10^{13} g/cm^3)	Broadband	Narrow band 1150 Hz
13.25	$\pm .10$	$\pm .10$
13.4	$\pm .11$	$\pm .11$
13.7	$\pm .24$	$\pm .30$

The deviation from point-particle inspiral is dominated by the **quadrupole tidal deformation** of each star.

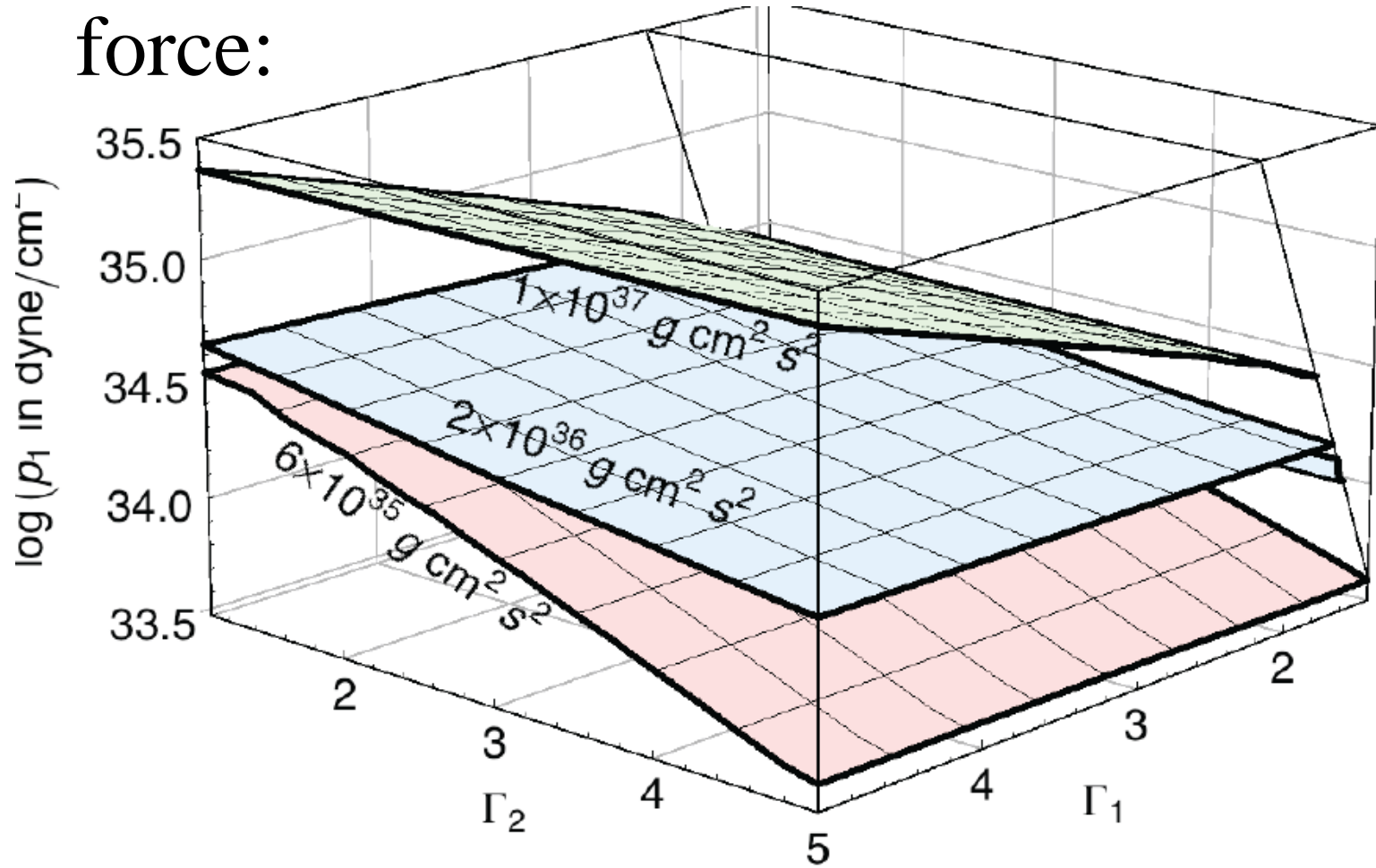
For large separations this is given by the Love number k : An external quadrupole gravitational field

$$E_{ij} x^i x^j$$

induces a quadrupole moment tensor

$$Q_{ij} = k R^5 E_{ij}$$

Surfaces of constant induced quadrupole moment (constant kR^5) for fixed tidal force:



(provided by Ben Lackey)

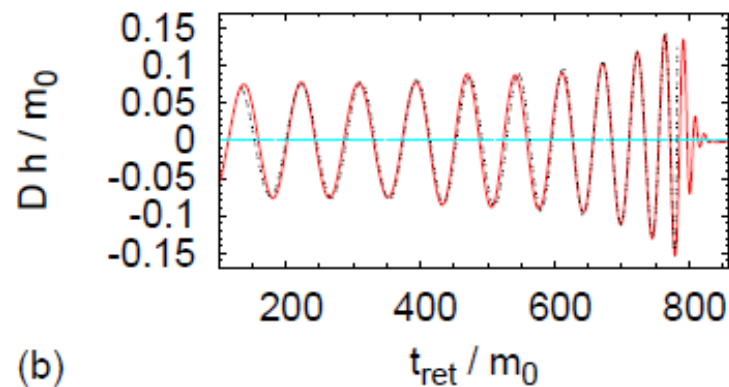
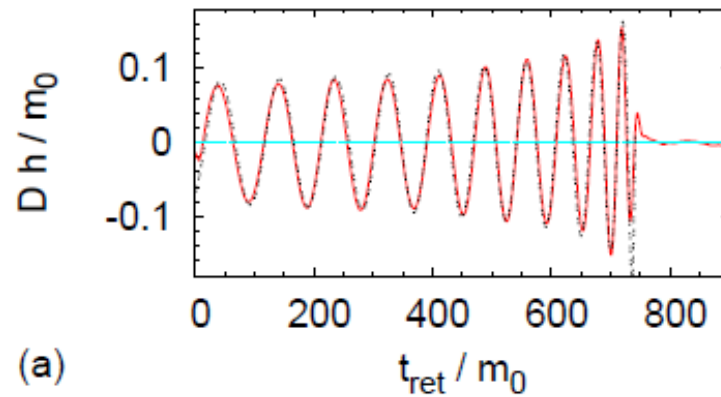
Black-hole – neutron-star inspiral

Again obtain surface in EOS space by
measuring departure of NS-BH
waveform from point-particle inspiral

A wide range of possible mass ratios, orbits, BH spins, implies a constraint surface thickened by uncertainty.

Shibata, (Kyoto)
Kyutoku,
Yamamoto (Tokyo)
Taniguchi (UWMn)

Waveforms for
nonrotating BHs,
mass ratios 2 and 5.



Whether tidal stripping (mass shedding) occurs depends on mass ratio and NS compactness

