Tailoring BKL to Loop Quantum Cosmology

Adam D. Henderson

Institute for Gravitation and the Cosmos
Pennsylvania State University

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Introduction

- Loop Quantum Cosmology: Big Bang singularity is replaced by Quantum Bounce in homogeneous & isotropic models.
- Singularity Resolution: Feature of full Loop Quantum Gravity or Artifact of Symmetry?
- Need to quantize more general models - Bianchi, Gowdy..
- Want simplified model encompassing large class of cosmological models with singularities.
- Growing numerical & analytical evidence that general cosmological singularities are described by the BKL conjecture.

Introduction

- BKL conjecture:
  - Spatial derivatives in the equations of motion become negligible near the singularity.
  - Most types of matter become negligible near singularity.
  - The dynamics near singularity is Mixmaster - sequence of Bianchi I separated by Bianchi II.
- Truncated theory obtained by neglecting spatial derivatives gives dynamics near the singularity.
- Quantizing this truncated theory may be one important step towards proving that general cosmological singularities are resolved.
- Need to describe the conjecture and the truncated theory in variables suited for quantization.

Outline

Preliminaries

- Phase Space: $(\widetilde{E}^a_i, K^i_a)$
    - $\widetilde{E}^a_i$ - invertible.
- $\{\widetilde{E}^a_i(x), K^j_b(y)\} = \delta^j_i \delta^a_b \delta^3(x-y)$
- Γ^i_a - Connection compatible with $\widetilde{E}^a_i$.

$$D_a \widetilde{E}^b_i + \epsilon_{ijk} \Gamma^j_a \widetilde{E}^{bk} = 0 \tag{1}$$

- Scalar, Vector, and Gauss constraints.

$$\widetilde{\widetilde{S}} \equiv -q\mathcal{R} - 2\widetilde{E}^a_{[i}\widetilde{E}^b_{j]} K^i_a K^j_b \tag{2}$$

$$\widetilde{V}_a \equiv 4\widetilde{E}^b_i D_{[a} K^i_{b]} \tag{3}$$

$$\widetilde{G}_{ij} \equiv -\widetilde{E}^a_{[i} K_{aj]} \tag{4}$$

Motivate Truncation

- What derivatives are negligible?
- Motivated by Scale-Invariant approach of Uggla, et al
  - Divide variables by trace of extrinsic curvature.
  - $K^{-1}e_i^a$ - Becomes degenerate at singularity.
  - $K^{-1}e_i^a$ suppresses spatial derivatives.
  - $K^{-1}e_i^a\partial_a\frac{Y}{K} \to 0$
- Supported by numerical simulations of Garfinkle.
- Problem: Inverses, K^{-1}, are difficult to quantize.
- Fortunately $\widetilde{E}_i^a$ has properties similar to $K^{-1}e_i^a$
  - Becomes degenerate at singularity, since $\sqrt{q} \to 0$
- Negligible derivatives would then be: $\widetilde{E}_i^a D_a \widetilde{Y}$

BKL Motivated Var.

BKL Variables

- $\widetilde{P}_{ij} = \widetilde{E}_i^a K_{aj} - \widetilde{E}_k^a K_{ak} \delta_{ij}$
- $\widetilde{C}_{ij} = \widetilde{E}_i^a \Gamma_{aj} - \widetilde{E}_k^a \Gamma_{ak} \delta_{ij}$
- $\widetilde{D}_i = \widetilde{E}_i^a D_a$

- $\widetilde{P}_{ij}$ and $\widetilde{C}_{ij}$ are densities with internal indices only.
- The constraints* and the EOM for $\widetilde{P}_{ij}, \widetilde{C}_{ij}$ can be written in terms of $\widetilde{D}_i, \widetilde{P}_{ij}, \widetilde{C}_{ij}, \underset{\sim}{N}$ only.
  - *Modified vector constraint: $\widetilde{\widetilde{V}}_i = \widetilde{E}_i^a \widetilde{V}_a$

BKL Conjecture

Conjecture

- Our form of the BKL conjecture in terms of these variables is:

1. $\widetilde{D}_i\widetilde{P}_{jk}$, $\widetilde{D}_i\widetilde{C}_{jk}$, and $\widetilde{D}_i\underset{\sim}{N}$ go to zero sufficiently fast as the singularity is approached.
    - Implies that $\widetilde{C}_{[ij]}$ also goes to zero.
2. $\widetilde{P}_{ij}$ & $\widetilde{C}_{ij}$ and $\underset{\sim}{N}$ remain bounded as singularity is approached.
3. Solutions to the full equations of motion are well approximated near the singularity by solutions to truncated equations obtained by setting derivative terms to zero.

Truncated Theory

- Obtain truncated theory by setting derivative terms to zero in EOM and constraints.

$$\widetilde{D}_i \widetilde{P}_{jk} = \widetilde{D}_i \widetilde{C}_{jk} = \widetilde{D}_i \underset{\sim}{N} = \widetilde{C}_{[ij]} = 0 \quad (5)$$

- Forms a subspace of the full phase space which is invariant under the full dynamics - Fixed Subspace.

- Truncation reduces full Poisson brackets to:

$$\{\widetilde{P}_{ij}, \widetilde{C}_{kl}\} = \widetilde{C}_{kj}\delta_{il} + \widetilde{C}_{jl}\delta_{ik} \quad (6)$$

$$\left\{\widetilde{P}_{ij}, \widetilde{P}_{kl}\right\} = \widetilde{P}_{kj}\delta_{il} - \widetilde{P}_{il}\delta_{kj} \quad (7)$$

Truncated EOMs

- Setting the derivative terms to zero we obtain the truncated constraints.

$$\widetilde{\widetilde{S}} = \widetilde{C}_{ij}\widetilde{C}^{ji} - \frac{1}{2}\widetilde{C}^2 + \widetilde{P}_{ij}\widetilde{P}^{ji} - \frac{1}{2}\widetilde{P}^2 \tag{8}$$

$$\widetilde{\widetilde{V}}_i = 2\epsilon_{jkl}\widetilde{P}^{kl}(\widetilde{C}_i{}^j - \widetilde{C}\delta_i{}^j) + 2\epsilon_{ijk}\widetilde{P}^{jl}\widetilde{C}_l{}^k \tag{9}$$

$$\widetilde{G}_{ij} = -\widetilde{P}_{[ij]} \tag{10}$$

- Constraints in terms of $\widetilde{C}_{ij}$ and $\widetilde{P}_{ij}$ only.

Truncated EOMS

- Truncated Equations of Motion:

$$\dot{\widetilde{C}}^{ij} = \underset{\sim}{N}[2\widetilde{C}^{(i}{}_{k}\widetilde{P}^{|k|j)} - \widetilde{P}\widetilde{C}^{ij}] \quad (11)$$

$$\dot{\widetilde{P}}^{ij} = \underset{\sim}{N}[-2\widetilde{C}^{ik}\widetilde{C}_{k}{}^{j} + \widetilde{C}\widetilde{C}^{ij}] \quad (12)$$

$$\dot{\widetilde{E}}^{a}_{i} = -\underset{\sim}{N}\widetilde{P}_{i}{}^{j}\widetilde{E}^{a}_{j} \quad (13)$$

- Evolution for $\widetilde{C}_{ij}$ and $\widetilde{P}_{ij}$ does not depend on $\widetilde{E}^a_i$.
- Evolution closed in terms of $\widetilde{C}_{ij}, \widetilde{P}_{ij}$
- Consistency: Truncated equations can be generated in two ways.
  1. Take the full equations of motion and set derivative terms to zero.
  2. Take Poisson brackets with the truncated constraints.

Gauge Fixing

- The Gauss and Vector constraint $\longrightarrow \widetilde{P}_{ij}$ and $\widetilde{C}_{ij}$ are symmetric and commute:

$$[\widetilde{C}, \widetilde{P}]_i{}^j = 0 \tag{14}$$

- Diagonalizing $\widetilde{P}_{ij}$ and $\widetilde{C}_{ij}$ gauge fixes Gauss & Vector constraints.
- Reduces to six degrees of freedom: (C_I,P_I) - I =1,2,3.
- Simple Poisson brackets

$$\{P_I, P_J\} = \{C_I, C_J\} = 0 \tag{15}$$

$$\{P_I, C_J\} = 2\delta_{IJ} C_J \tag{16}$$

Reduced Phase Space

- Hamiltonian Constraint:

$$\frac{1}{2}\left(\sum_I C_I\right)^2 - \sum_I C_I^2 + \frac{1}{2}\left(\sum_I P_I\right)^2 - \sum_I P_I^2 = 0 \qquad (17)$$

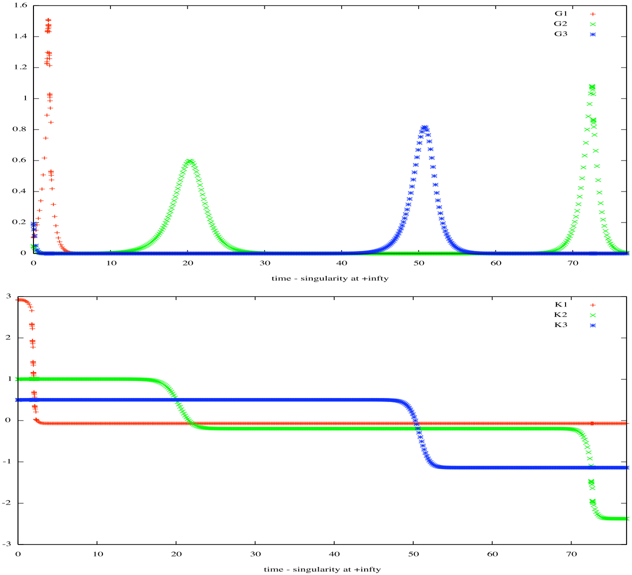
- Evolution Eqns:

$$\dot{P}_I = \underset{\sim}{N} C_I (\sum_J C_J - 2C_I) \qquad (18)$$

$$\dot{C}_I = -\underset{\sim}{N} C_I (\sum_J P_J - 2P_I) \qquad (19)$$

Dynamics of Reduced System

- Mixmaster dymamics seen analytically & numerically.
- Numerical evolution →
- u-map derived analytically.



Conclusion

- Described the BKL conjecture in a Hamiltonian formulation with variables suitable for quantization.
- Consistent truncation of the full theory obtained by setting $\widetilde{E}_i^a D_a$ terms to zero
- Truncated theory reproduces expected Mixmaster behavior.
- Would like to quantize truncated theory.
  - Problem: Constraints in terms of $(\widetilde{P}_{ij}, \widetilde{C}_{ij})$ or $(\widetilde{E}_i^a A_a^j, \widetilde{C}_{ij}$ or $\widetilde{P}_{ij})$
  - Need to write $\widetilde{E}_i^a K_a^j$ or $\widetilde{E}_i^a \Gamma_a^j$ in terms of holonomies and fluxes.
- Possible intermediate step - Effective equations.