Towards Scale-invariant Cyclic universes

Tirthabir Biswas

IGC, Penn State University, USA

arXiv:0801.1315 [hep-th]
arXiv:0707.4679 [hep-th] with S Alexander & R Brandenberger

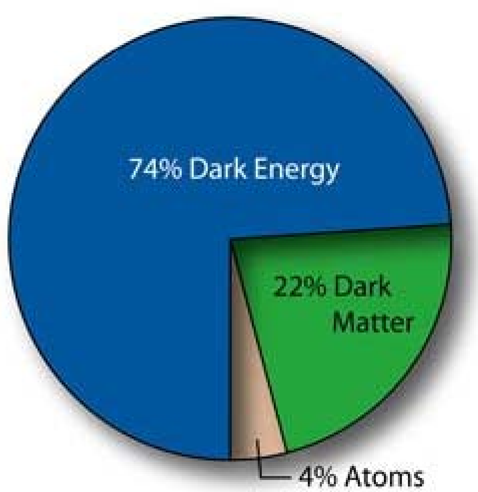
Standard Model of Cosmology

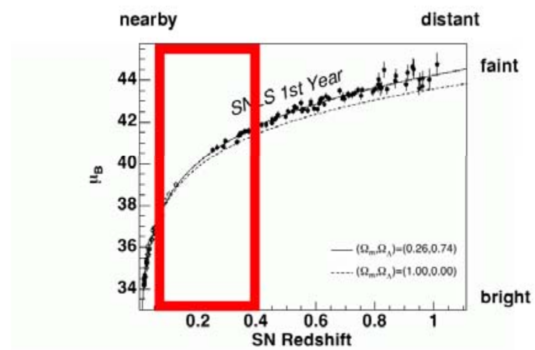
$$ds^2 = a^2(\tau)[-(1+\Phi(\tau,x))d\tau^2 + (1-\Phi(\tau,x))dx^2]$$

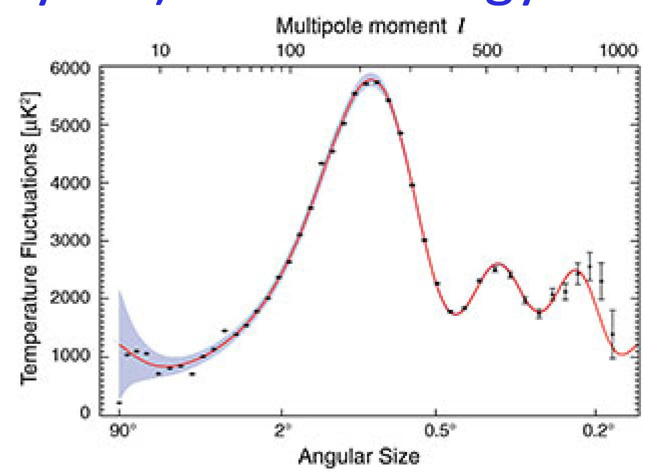
Key ingredients $\qquad k^3 P_\Phi(k) \approx const$

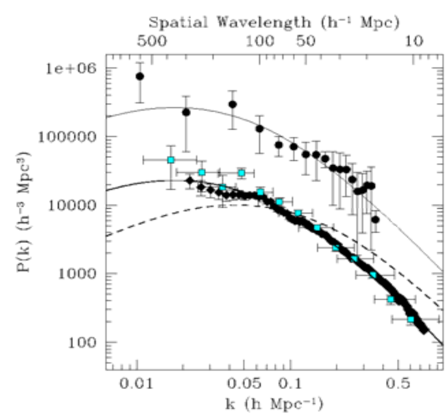
- GR+Homogeneous Isotropic cosmology+ Perturbations
- Inflation →Radiation → Matter(DM+baryons)→Dark Energy

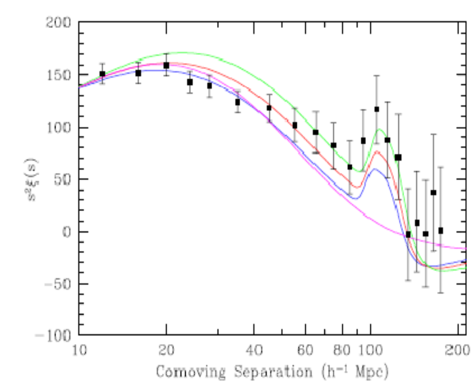
Concordant ΛCDM model











Why do we still have our jobs?

Finally we know for sure that we almost know nothing

Inflaton, Baryons, Dark Matter, Dark Energy????

Two Attitudes

- SM is fine, try to address the hard questions
- Search for alternative Non-singular (QG) cosmology
- Cosmology is an ultimate magnifier

1. Length

$$H_0^{-1} \xrightarrow{\text{radiation}} \frac{T_0}{T_{GUT}} H_0^{-1} \xrightarrow{\text{inflation}} e^{-N} \left(\frac{M_p^2}{T_{GUT} T_0} \right) l_p = e^{-(N-75)} l_p$$

2. Energy density can become Plackian
3. Spatial curvature can become Planckian

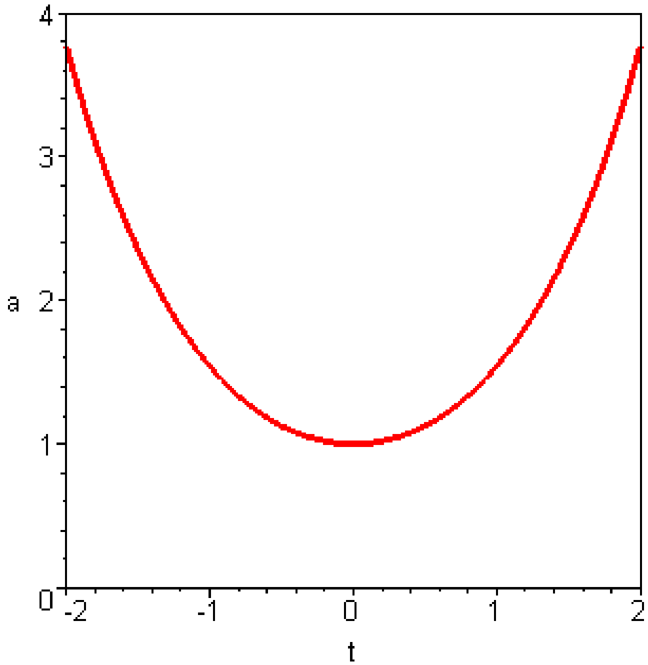
- Observable Quantum Gravity Effects may be testable already (WMAP) and/or near future (Planck)

1. Spectrum
2. Gravity waves
3. Non gaussianity and non adiabaticity

Non-singular Cosmologies

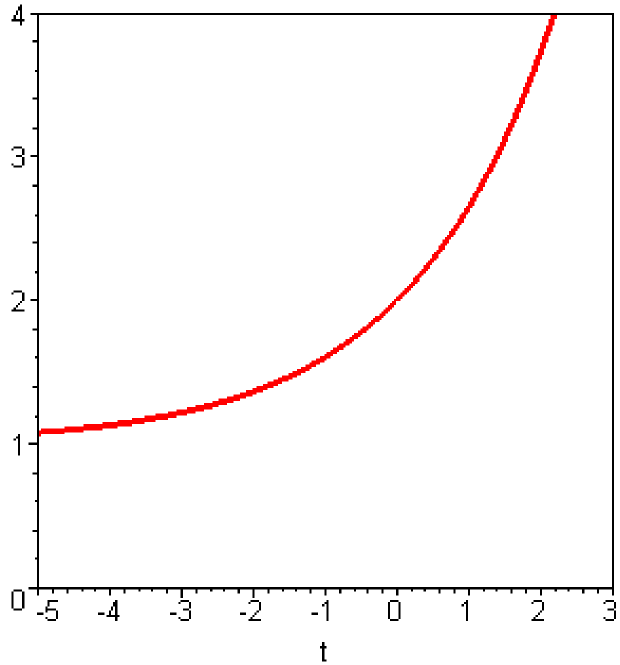
- “Effective” FLRW cosmologies a good description: LQC + BKL
- Look into the “eternal past”

$$R \sim H^2 \sim (\dot{a}/a)^2$$



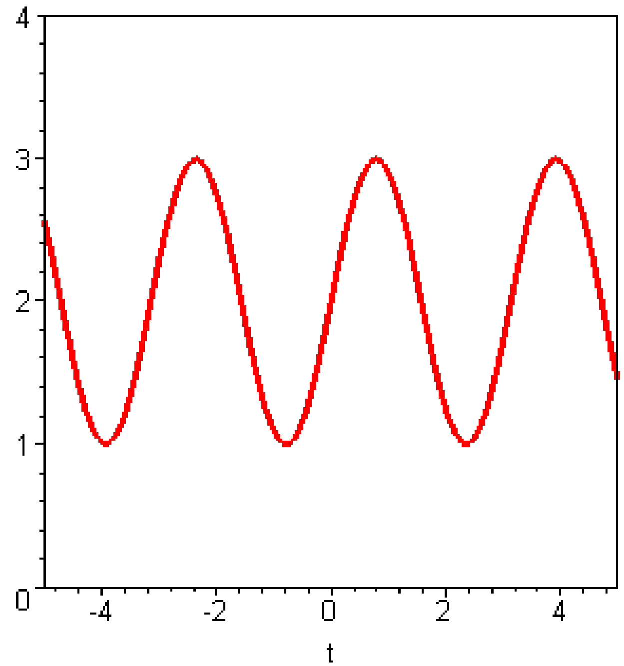
Bouncing Universe

Novello,Salim; Melnikov,Orlov,70’s
Gaperini, Maggiore & Veneziano
(pre-big bang) ’97



Emergent Universe

Ellis & Maartens ’04



Cyclic Universe

Einstein,Freedman,Tolman,Lemaitre, 30’s
Bondi,Gold,Narlekar,Hoyle (steady state) 50’s
Steinhardt & Turok (ekpyrotic), ’02
Freese et.al.; Frampton & Baum (phantom)

Plan of the talk

- Virtues and Challenges of cyclic cosmology
- Tracking perturbations around Bounce
- New “emergent cyclic Universe”
- Cyclic Inflation

Challenges & Virtues of Cyclic Universes

I. **Nonsingular geodesically complete Universe**

- Consistent (ghostfree) Bounce
- LQC
- p-adic/SFT inspired Non-local modifications of gravity [Seigel, Mazumdar, YT]
- BCS Gap energy: [Alexander, & YT]
- Coupling of fermions to gravity $\Rightarrow$ four fermion interaction
- Attractive $\Rightarrow$ negative energy required for bounce
- Gap energy $\rightarrow$ chemical potential $\rightarrow$ number density $\rightarrow$ volume
- Nontrivial volume dependence can temporarily violate DEC

II. Turn-around

- Spatial curvature (no Dark Energy)
- Scalars, phantom

III. Classic puzzles:

- Horizon: All
- Flatness, Largeness, Entropy: Emergent cyclic models
- ? Homogeneity/Isotropy: Some aspects we will be able to address

IV. Black hole over-production & Dark energy

- Quintessence (ekpyrotic) makes BH's dilute
- Phantom makes BH's disintegrate

V. ? Generating Scale-invariant Perturbations

- Scalar field fluctuations [Steinhardt et.al]
- Stringy thermal fluctuations [Brandenberger et. al]
- "New" ideas

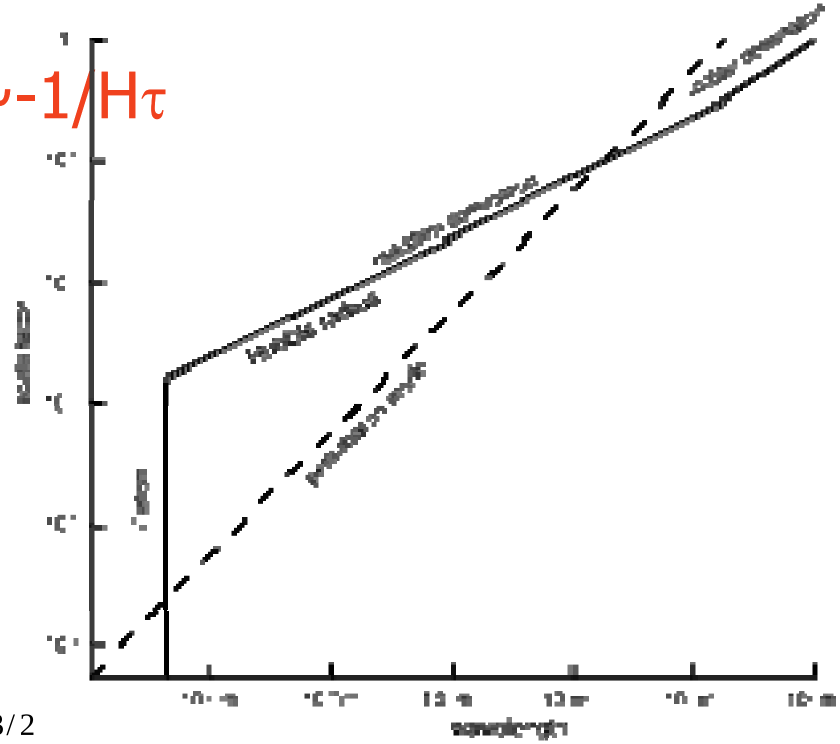
VI. Tolman's Entropy or the "Super Big Bang" Problem

- Thermal UV phase (Hagedorn phase) exists +
- BB singularity problem is solved

Transferring fluctuations via bounce

Inflation Basics   V''(Φ)≈0, a~-1/Hτ

$$\sigma_k'' + \left(k^2 - \frac{a''}{a}\right)\sigma_k = 0 \qquad \sigma \equiv \frac{\phi}{a}$$



- Sub-Hubble
- $H << k \quad \Rightarrow \quad \sigma_k \sim k^{-1/2} e^{ik\tau}$
- Super-Hubble

  $H >> k/a \qquad \sigma_k \sim A(k) a(\tau)$
- Matching at Hubble crossing

  $$H = k/a \quad \Leftrightarrow \quad k \sim 1/|\tau| \qquad \varphi_k \sim k^{-3/2}$$
- Power Spectrum

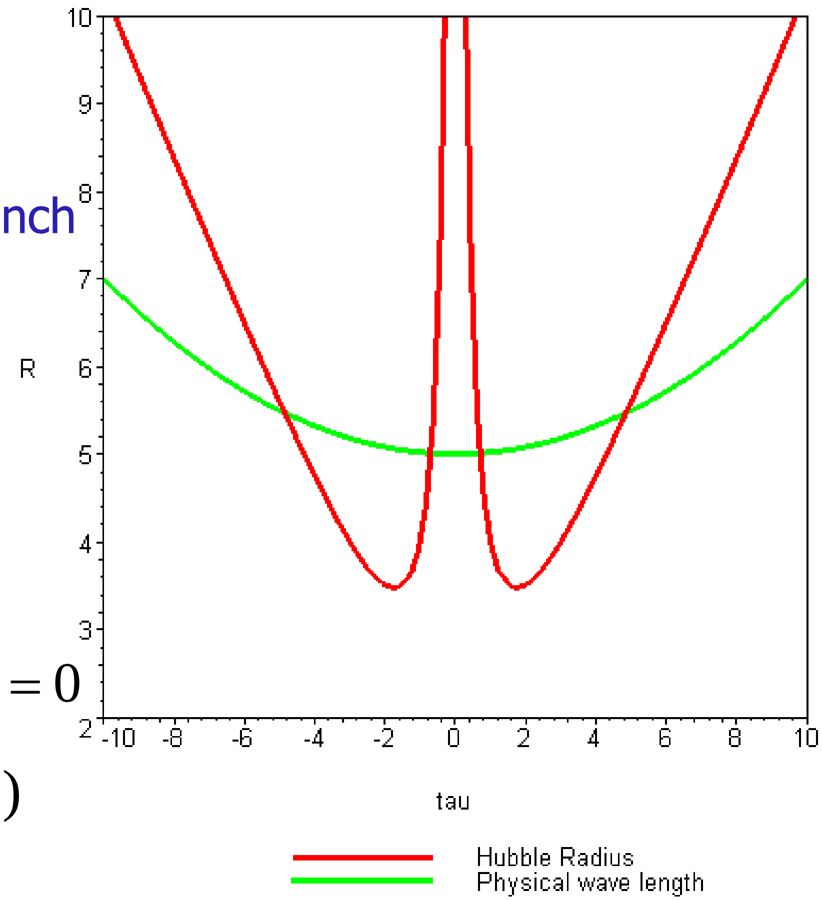
  $$P_\phi \sim k^3 |\varphi_k|^2 \sim const.$$

Ekpyrotic

- Generates scale-invariant spectrum in the "growing mode" during contraction phase.

Perturbations in Nonsingular Bounce

- Interested in constant mode in expanding branch
- Mode matching ambiguities
- Physics at bounce not known/singular
- Numerical solutions tricky



How to avoid the pit-falls?

- GR equation valid away from bounce

$$\Phi_k'' + 3(1+\omega)\mathcal{H}\Phi_k' + [\omega k^2 + 2\mathcal{H}' + (1+3\omega)\mathcal{H}^2]\Phi_k = 0$$

Solutions known: $(-\infty, \tau_p)$ & (τ_p, ∞)

- In super-Hubble spatial derivatives not important
  - If no inflation $k_{ph} << \tau_p^{-1} \sim O(M_p)$
  - Near bounce also like super-Hubble , even if H $\rightarrow 0$ $H' << \tau_p^{-1}$
  - Higher order spatial derivatives can also be neglected
  - Perturbation eqn. only depends on time, $(-\tau_k, \tau_k)$
  - find fluctuations for scale-factor $\Leftrightarrow$ only need homogeneous isotropic cosmology
- Matching in overlapping regions $(-\tau_k, -\tau_p)$ & (τ_p, τ_k)

An example

Assumption

- Perturbation equation not effected by new physics
- Non-local physics changes global evolution
- Does not effect perturbations
- Casimir Energy, Gap energy

Results

- Depends on the new physics (background)
- Generally modes mix! Good news for ekpyrotic scenarios
- Modes can even switch
    - Modes switch at $\omega \approx 5.314$,
    - Mixes when $\omega >> 1$ (ekprotic limit)

Emergent Cyclic Universe

Tolman's Entropy Problem

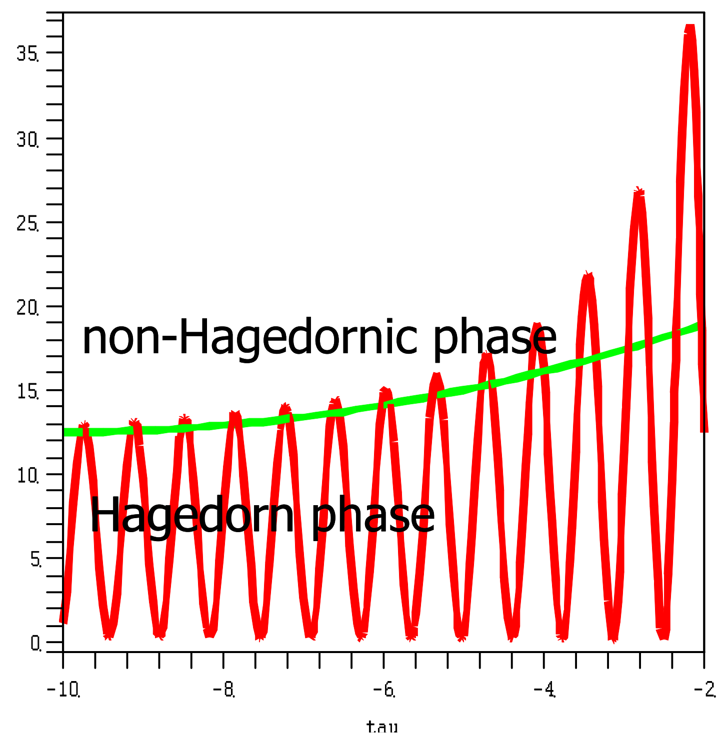
- ➢ Entropy is monotonically increasing
- ▪ Universe atmost quasi-periodic
- ▪ Entropy (Energy, period) vanishes in a finite time in the past
- ▪ Beginning of time – back to square 1

- ➢ Thermal Hagedorn Phase, $T=T_H \sim M_S$
- ▪ All string states in thermal equilibrium
  entropy constant
- ▪ Below critical temperature, massive states
  decouple, entropy produced.
- ▪ As cycles shrink, universe is hotter
  less time in entropy producing

  more time in

$$\Delta S \to 0 \qquad \Rightarrow S_n \to S_{-\infty} \neq 0$$

- ▪ Cycles asymptote to a periodic evolution



Stringy Toy Model

- Hagedorn half $H^2 = \frac{1}{3}[\rho_{hag} - \rho_{cas} + \rho_{curv}] = \frac{1}{3}\left[\frac{S}{a^3} - \frac{\Omega_c}{a^4} - \frac{\Omega_k}{a^2}\right]$
  - Eternally periodic universe

$$a(\tau) = \frac{S - \sqrt{S^2 - 4\Omega_c\Omega_k} \cos \nu\tau}{2\Omega_k} \qquad \nu \equiv \frac{T_H^2}{M_p}\sqrt{\frac{\Omega_k}{3}}$$

- Non-Hagedorn half (no energy exchange)
  - Hagedorn matter = massless (r)+massive (m)
  - $S \to \Omega_m$ & $\Omega_c \to \Omega_c - \Omega_r$ $\Rightarrow$ $a_{\max} \sim \sqrt{\Omega_r}$
- Gluing the two halves (Ω_r, Ω_m, a_{tran})
  - Entropy is conserved
  - Matter and radiation was in equilibrium till the transition
  - Phenomenological Input: $\mu \equiv \frac{\rho_m}{\rho_r} \sim 10^{-22} << 1$

$$\Omega_r \sim S^{4/3} \Rightarrow a_{\max} \sim S^{2/3} \qquad a_{tran} \sim S^{1/3}$$

Energy Exchange & Entropy Production

- Matter coverted to radiation [Tolman,Barrow et.al.]

$$\dot{\rho}_r + 4H\rho_r = T_H^4 s \qquad \dot{\rho}_m + 4H\rho_m = -T_H^4 s$$

$$\dot{S}_{tot} = a^3 s\left(\frac{T_H}{T_r} - \frac{T_H}{T_m}\right)$$

- Consistent with 1st & 2nd law of thermodynamics
- Conserves total stress-energy tensor
- Breaks time-reversal symmetry – arrow of time
- Exchange function, s(a,Ω)
- Small cycle limit, s ~ const.

$$S = S_{cr}\left[1 + \frac{1}{Cn^2}\right]$$

Crucial Difference

- Singular bounce ➡ Γ interaction cannot keep up with H → ∞
- Thermal equil cannot be mantiened ➡ entropy production during bounce
- Nonsingular bounce ➡ H is finite, a thermal Hagedorn phase can exist.

Cyclic Inflation

Can we mimick inflation?

- Entropy production by decay of massive particles:

$$\left(\frac{S_{n+1}}{S_n}\right)=\left(\frac{S_r}{S_m}\right)\sim\frac{\rho_r^{3/4}V}{\rho_m VM^{-1}}\sim\frac{T_H}{T_d}\equiv\kappa$$

- Entropy increases by a constant factor
- If decay time > Period, κ smaller
- So does the scale factor!
- Energy density remains same
- Inflation over many many cycles

$$a_{tran}\sim S^{1/3}\Rightarrow\frac{a_{n+1}}{a_n}\sim\kappa^{1/3}$$

- Massless scalar field will see inflation on an average, if

$$\tau H_{av}<<1$$

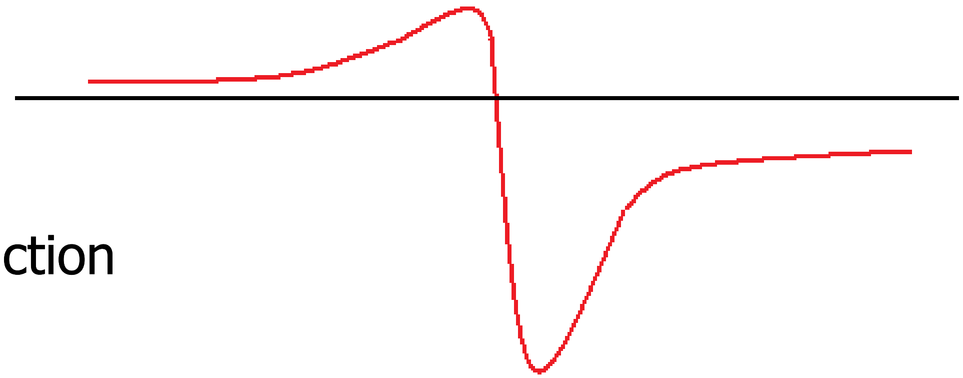
More detail requirements

- With curvature turnaround τ increases
- -ve consmological constant $\sim -\lambda^4 \Rightarrow \quad \tau \sim \frac{T_H}{\lambda^2} \quad \lambda < T_H$

$$H_{av} = \frac{\int H dt}{\int dt} = \frac{\ln\left(\frac{a_{n+1}}{a_n}\right)}{\tau} \Rightarrow \ln\left(\frac{a_{n+1}}{a_n}\right) = \frac{\ln \kappa}{3} << 1$$

- Over many many many cycles we realize inflation

How to exit?



- Introduce a potential
- Kinetic energy at end of contraction depends on slope:
- Total energy can go from – ve to + ve
- Can zoom past the minimum and into + ve potential region
- Can even mimick quintessence, not necessary.

Thermal fluctuations

Spectrum [Peebles'93, Pogosian & Magueijo]

$$\delta_L^2 = \frac{\Delta E^2}{E^2}\bigg|_L = \left(\frac{d^2 \ln Z}{d\beta^2}\right) \bigg/ \left(\frac{d \ln Z}{d\beta}\right)^2 = \frac{T^2}{\rho^2 L^3}\frac{\partial \rho}{\partial T} \sim \frac{1}{(TL)^3}$$

- Mechanism
  - Energy fluctuation in a given volume related to Power spectrum
  - White noise spectrum $\delta_L^2 = \int dk k^2 W(kL) P(k) \Rightarrow P(k) = \delta_k^2 = \frac{T^2}{\rho^2}\frac{\partial \rho}{\partial T} k^0 \to \frac{4}{g} T^{-3}$
  - Thermal correlations only for sub-Hubble modes
  - Hubble Crossing $\frac{k}{a} \sim H \sim \frac{T^2}{M_p} \Rightarrow k \sim T \Rightarrow P(k) = k^{-3}$
  - Conformal radiation leads to scale invariance
  - P(k) transferred to gravity,$P_\Phi(k)$ $\nabla^2 \Phi \approx \delta\rho \Rightarrow k^2 \Phi_k = a^2 \rho \delta_k \sim a^2 H^2 \delta_k$
  - Super Hubble governed by Φ_k
- Robust and general mechanism (no fine-tuning):
  - At Hubble crossing validity of GR
  - Radiation domination
  - Contraction so that modes leave the Hubble radius
  - No mode mixing, Φ_k remains constant

Amplitude

- The problem for symmetric bounce
- Consider scale which enters Hubble radius at T_{eq} (1Mpc)

$$\delta_L^2 = \frac{1}{g_*(LT_L)^3} \sim \frac{1}{g_*}\left(\frac{H_L}{T_L}\right)^3 \sim \sqrt{g_*}\left(\frac{T_L}{M_p}\right)^3 << 10^{-8} \rightarrow \quad T_L \sim 10^{13} Gev$$

- Exactly the right scenario to “amplify” perturbations

$$k = T_L^2 a_L = T_{eq}^2 a_{eq} \qquad \& \qquad S \sim (Ta)^3 \sim T^{-3}$$

$$\left(\frac{S_0}{S_{-1}}\right) \equiv \kappa = 10^{67}$$

- a_p is increasing λ_{ph} is decreasing with cycles

$\Rightarrow$ universe has to expand more for the same mode to exit

we need

- Thermal Equilibrium in Hagedorn phase [Frey et.al, Hindmarsh & Skliros]
- Small cycles: Thermal equilibrium to be mantained $H < \Gamma_{\text{int}}$

$$\Gamma_{\text{int}} \sim T_H g_s^2 n \qquad H \sim T_H^2 / M_p$$

$$T_H \sim 10^{-3} M_p \qquad g_s^2 > 10^{-5}$$

- Large cycles: $H \propto S^2$

  Thermal equilibrium around bounce short lived

  Near bounce large amounts of entropy produced
- Entropy Production around Bounce
- Naïve Estimate: when radiation converts to Hagedorn matter

$$\left(\frac{S_0}{S_{-1}}\right) = \left(\frac{S_H}{S_r}\right) \sim \frac{\rho_H V T_H^{-1}}{\rho_r^{3/4} V} \sim \frac{\rho_b^{1/4}}{T_H} \sim S_{-1}$$

- Holographic Saturation: Can't trust physics at super-Planckian energy densities

  Treat bounce at black box, use holographic entropy bound

  If saturated we get the same estimate!

$$S \leq area \sim \left(\frac{E}{M_p}\right)^2 \Rightarrow S_0 \leq S_{-1}^2 \left(\frac{T_H}{M_p}\right)^2$$

$$S_0 \sim 10^{134} \quad \& \quad S_{-1} \sim 10^{67}$$

Dark Energy and Black Hole Problem

- Add Λ ~ (mev) there exists a last (our) cycle! DE phase

- Previous cycles very short
  - no matter domination, no LSS
  - no junk (BH/inhomogeneities) from previous cycle
  - thermal density fluctuation very small

- Toy Model can be extended to ekpyrotic/phantom type late cycles

Summary

The story so far?

- Began as string size, curved universe, in almost periodic & almost Hagedornic phase: emerging phase
- (a) Constant cycle "inflationary" phase -> graceful exit to long lived cycle
- (b) With entropy production, Hagedorn phase becomes shorter,
- large entropy production starts
- Universe highly asymmetric and large
- Very large cycles (like ours), curvature gives way to Dark Energy

Problems addressed

- Singularities: BBS & super BBP
- Classic Problems: horizon, flatness, entropy, largeness
- Late time: inhomogeneity/BH overproduction & DE
- Perturbations: spectrum and amplitude

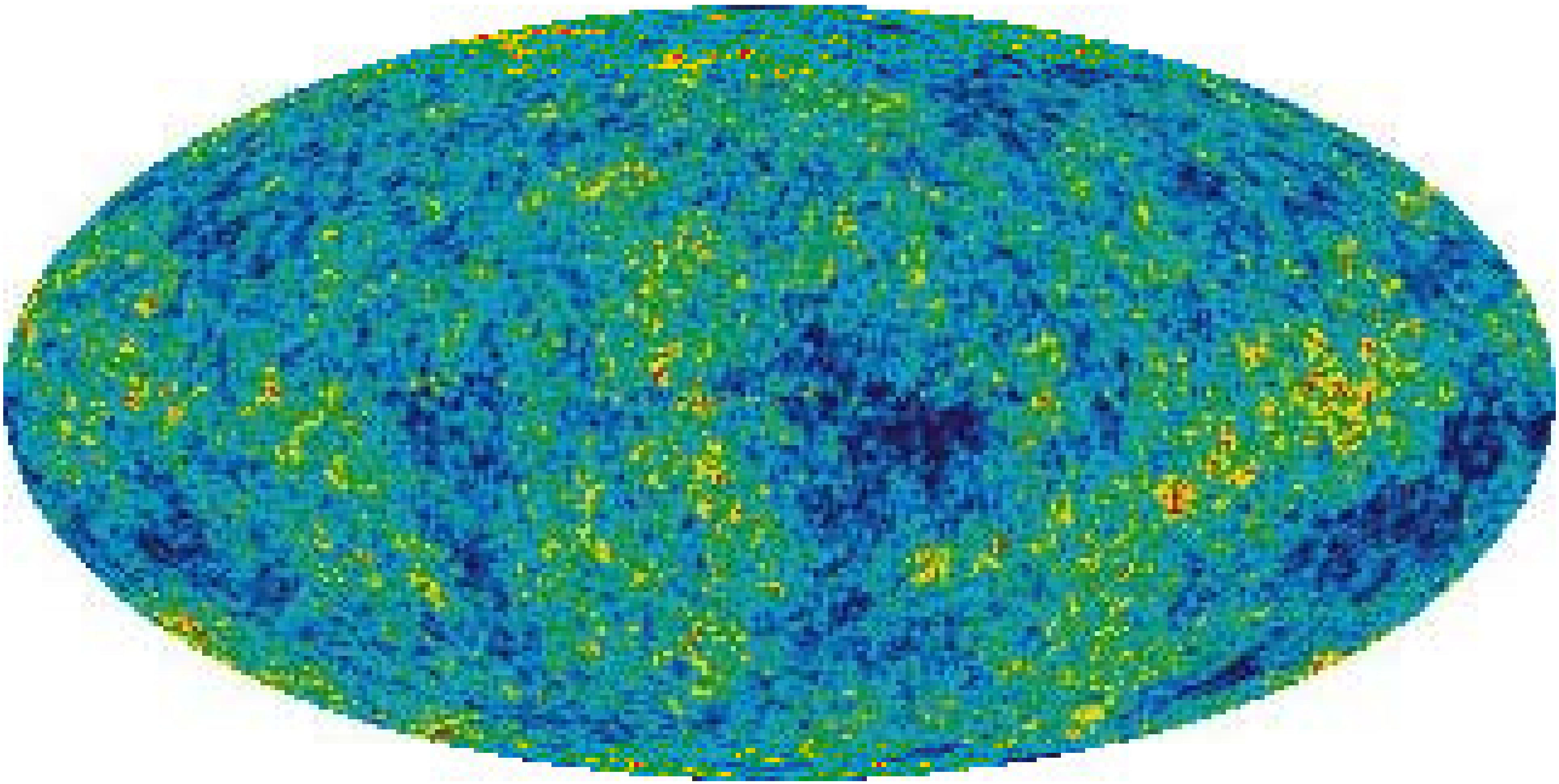
Needs investigation

- Initial homogeneity/isotropy, Mix master behaviour
- Hagedorn physics: Non-equilibrium dynamics, Casimir Energy, Decay rate…
- Mode mixing during Bounce? Is Φ really constant during bounce?

Predictions/Tests

- Gravity Waves, Non-gaussianity…

Precision Cosmology



Cosmic Microwave Background

- We need an asymmetric bounce, the comoving scale has to exit the Hubble radius much earlier.
- This is precisely what we have via entropy production

Quantum Gravity: A toy Model

Motivation

- Stringy
- Dual Field theory action for strings on Random Lattice

$$\hat{S} = \int d^D x \; tr \left[\frac{1}{2} \phi e^{-\alpha' \Box /2} \phi + G^{n-2} \phi^n \right]$$

Linear Regge trajectories: Confinement [Grisaru, Siegel, Y.T.]

- Tachyons in open SFT and p-adic string theory has similar form
- Higher Derivative but Ghost free

$$S = \int d^4 x \; \phi \Box (\Box + m^2) \phi \Rightarrow \Box (\Box + m^2) \phi = 0$$

$$\Delta(p^2) = \frac{1}{p^2 (p^2 + m^2)} \sim \frac{1}{p^2} - \frac{1}{(p^2 + m^2)}$$

- Non-singular UV & IR behavior

Weinberg's "Asymptotic safety"

- Non-perturbative Quantum gravity

Close to Planck scale, all terms important

Model

- Action $$S = \frac{M_p^2}{2}\int d^4x\sqrt{-g}\left[R + \sum_{n=0} c_n R \Diamond^n R\right] \qquad c_n \sim \frac{1}{M^{2n+2}}$$

- Generalized ``Einstein's" Field Equations

$$\tilde{G}_{\mu\nu} \equiv G_{\mu\nu} + \sum_{n=0}^{\infty} G^n_{\mu\nu} = T_{\mu\nu}$$

$$G^n_{\mu\nu} \sim c_n(\Box^{n+1}R + \Box^p R \Box^m R)$$

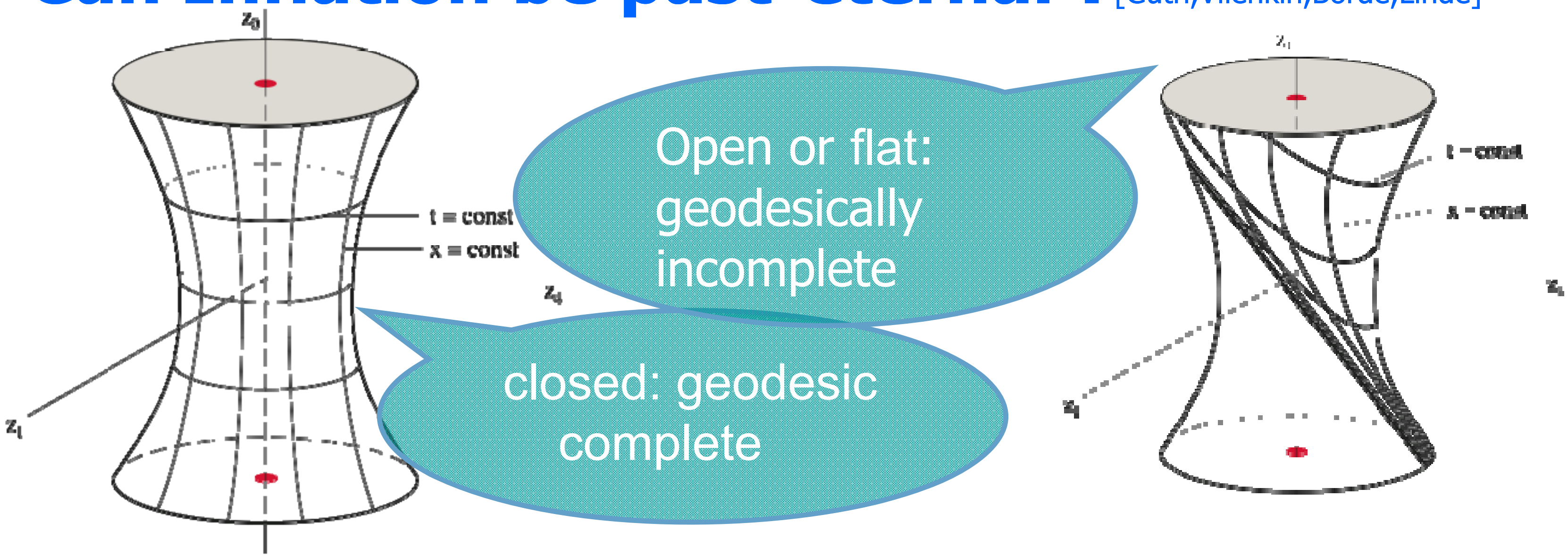
- Conservation Equation $\nabla^\mu \tilde{G}_{\mu\nu} = 0$

For Cosmology $\tilde{G}_{00} = 0$ equation suffices

- Exact Bouncing solutions! a(t) = cosh(λt)

$$\Lambda \neq 0, \qquad \rho_{rad} \sim M_s^2 M_p^2 \qquad \& \quad \lambda \sim M_s$$

- **Can Inflation be past-eternal ?** [Guth,Vilenkin,Borde,Linde]



- **Big Bounce:** H = 0 **&** $\ddot{a} > 0$

$$H^2 = \frac{\sum \rho_I}{3M_p^2}$$

- Flat/Open: DEC,WEC violation

Plan for the rest of the Talk

- Consistent (ghostfree) modification of Gravity
  LQC, Bouncing and cyclic Universes
  Non-perturbative gravity $\Rightarrow$ Bouncing Universe
- Consistent (ghostfree) modification of Matter-sector
- Ghost Condensation [Arkani-Hamed et.al.,Khoury et.al.]
- Casimir Energy and an "emergent cyclic Universe"
  Hagedorn Physics & Tolman's Entropy Problem
- Gap Energy and cosmological BCS Condensation [Alexander & Vaid]

- Generating Scale-invariant Perturbations
- Scalar field fluctuations [Steinhardt et.al]
- Stringy thermodynamic fluctuations [Brndenberger,Nayeri & Vafa]

Thermodynamic Fluctuations during Hagedorn Phase

Energy to Power Spectrum

- Energy to density fluctuations:

$$|\delta\rho_k|^2 \sim k^3 |\delta E(r \sim k^{-1})|^2$$

- Energy fluctuations from Heat Capacity

$$Z \sim \sum e^{-\beta E} \qquad \Rightarrow \delta E(r) \sim T^2 C_v \sim \frac{T}{T_H - T} r^2$$

- Matter fluctuations to metric perturbations

$$\nabla^2 \Phi = 4\pi G \delta\rho \qquad \Rightarrow |\Phi_k|^2 \sim k^{-4} |\delta\rho_k|^2$$

- Power Spectrum

$$P_\Phi \sim k^3 |\Phi_k|^2 \sim \frac{T}{T_H - T}$$

CMB Spectrum: Minimal Requirements

- Fluctuations should come from massive modes and not massless (radiation)

$$C_{rad} << C_{massive} \qquad \Rightarrow \frac{\Delta T}{T_H} < 10^{-30}$$

- Amplitude:

$$\delta^2_{CMB} \sim 10^{-10} \sim \left(\frac{M_s}{M_p}\right)^4 \frac{T_H}{\Delta T}$$

$$\frac{T_H}{\Delta T} = 10^{30} \qquad \Rightarrow \frac{M_s}{M_p} \sim 10^{-10}$$

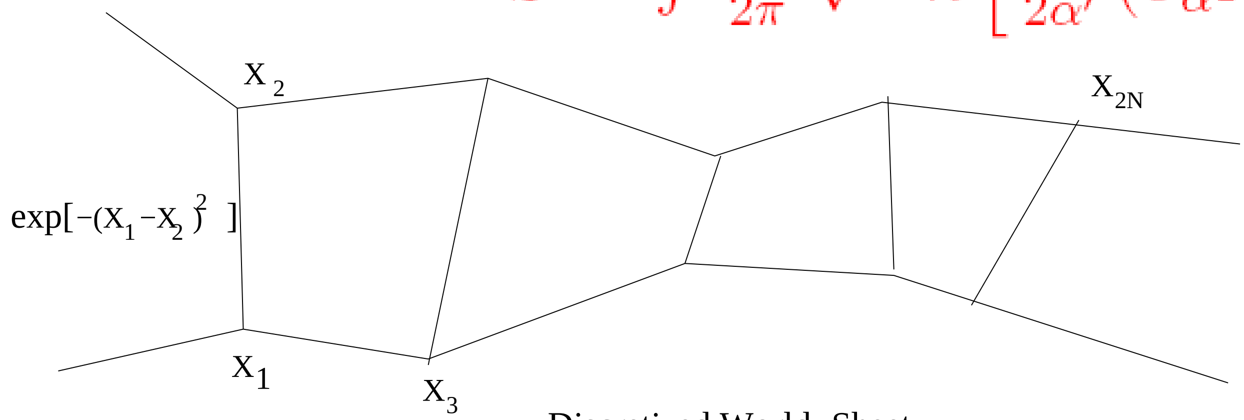
- Spectral tilt:

$$|\eta_s - 1| \approx 10^{-60}\left(\frac{M_s}{\lambda}\right)^2 \frac{T_H}{\Delta T}$$

typically one obtains almost perfect scale-invariance!

- **t' Hooft dual to string theory**
- Polyakov action:

$$S = \int \frac{d^2\sigma}{2\pi}\sqrt{-h}\left[\frac{h^{\alpha\beta}}{2\alpha'}(\partial_\alpha X)(\partial_\beta X)\right]$$



Discretized World−Sheet

- Strings on Random lattice [Douglas,Shenker] $S = \sum_{ij}(X_i - X_j)^2$

$$\Rightarrow Z = \int \mathcal{D}h\, \mathcal{D}X\ e^{-S} = \sum \int d^D X\ \prod_{ij} e^{-\frac{1}{2\alpha'}(X_i - X_j)^2}$$

- Dual Field theory action

$$\hat{S} = \int d^D x\ tr\left[\frac{1}{2}\phi e^{-\alpha'\Box/2}\phi + G^{n-2}\phi^n\right]$$

Linear Regge trajectories: Confinement [Grisaru, Siegel, Y.T.]

Finite Order Gravity

Improved UV behaviour: 4th Order Gravity

$$S = \int d^4x \, \sqrt{-g}(R + c_0 R^2 + b_0 C^2)$$

even Renormalizable [Stelle, 1978]

Asymptotically free + Renormalizable!

Unfortunately $b_0 \neq 0 \Rightarrow$ Ghosts

If $b_0 = 0$ Asymptotic freedom, Renormalizability lost

- (Ghost + assymptotically) free gravity => NP gravity

Propagator

- Scalar-Tensor Picture: HD terms in φ

$$S = \int d^4x\sqrt{-g}\left[e^{-\phi}R + \psi\sum_0^\infty c_i\Box^i\psi - \{\psi(e^{-\phi}-1)\}\right]$$

p-adic scalars in a curved background + dilaton?

- Field Equation

$$\left(1 - 6\sum_0^\infty c_i\Box^{i+1}\right)\phi \equiv \Gamma(\Box)\phi = 0 \Rightarrow \Delta(p^2) = \frac{1}{\Gamma(-p^2)}$$

- Ghost free if Γ(□) has:

  a single zero, R^2 gravity $\quad \Delta(p^2) = \frac{1}{(p^2+m^2)}$

  no zeroes, Gaussian's $\quad \Delta(p^2) = e^{-p^2/m^2}$

- Improved UV behaviour: $\quad h \sim \frac{\mathrm{erf}(r)}{r}$

Transition to FRW, $\Lambda = 0$

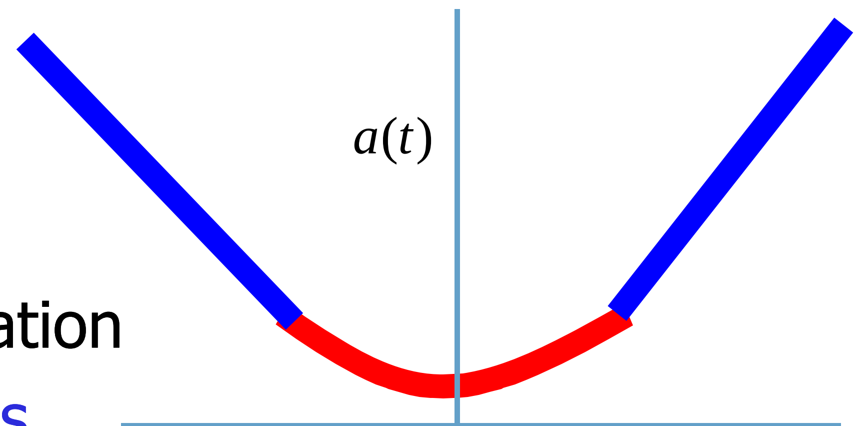
Late times

$a(t) \to e^{\lambda t}$ & HD terms $\to$ sech$^2(\lambda t) \sim e^{-2\lambda t} \to 0$

=> Einstein Gravity & dS Universe => $\Lambda \neq 0$

Near Bounce

$G_{00} \to 0$ but HD terms finite



Approximate Bounce

- Small times: HD terms = radiation

  We found ghost free examples
- Transition: HD terms ~ G_{00}
- Large times: FRW cosmology, HD terms << 0

$$a(t) \sim t^{1/2}, \qquad G_{00} \sim \frac{1}{t^2}, \qquad \tilde{G}_{00}^n \sim \frac{1}{t^{2(n+1)}}$$

Asymptotic Safety [Weinberg, 1976]

- Renormalizability replaced by asymptotic safety
- Quantum behavior captured by RG flow

$$\mu \frac{dg(\mu)}{d\mu} = ag^2(\mu) \qquad (a>0) \qquad g(\kappa\mu) = \frac{g(\mu)}{1-ag(\mu)\ln\kappa}$$

- Asymptotic safety = non-singularity → UV fixed point

  4 d gravity => $G_N \to 0$ asymptotic freedom

- Although ghost free finite HD gravity theories exist,

  (Ghost + asymptotically) free gravity => NP gravity

- Quantum Gravity actions (closed under renormalization flows) contains specific infinite series of HD terms:

  [Krasnov, gr-qc/0703002]

  Equivalent to “second order theory”, no IVP

Hagedorn Phase

Qualitative Behaviour

- Close to $T=T_H \sim M_S$ massive (winding) string states are excited. Pumping energy doesn't increase temperature, produces new states.
- Thermodynamics no longer determined by massless modes: $$E = T_H S - bVT_H{}^{d+1} + \ldots$$
- Cosmological Evolution: Entropy is constant

  Energy, Temperature remains approximately constant

$$\frac{\Delta T}{T_H} \equiv \frac{T_H - T}{T_H} \sim \exp\left[-\frac{E}{V^{2/3}T_H^3}\right]$$

- Transition to radiation occur when $S \sim VT_H{}^d$

Cosmological BCS Condensation

- BCS Theory
- Free theory, fermions filled upto Fermi-sea
- Attractive four-fermion coupling contributes to negative energy.
- Vacuum gets a negative shift with the formation of mass-gap.

$$L = -i\overline{\Psi}\gamma^{\mu}\partial_{\mu}\Psi - -$$

- Auxillary field
- Integrate the fermions, use mean field theory to get non-perturbative potential for Delta
- Gap Equation

Potential

- Trace equation (Lorentz gauge) $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\Rightarrow \widetilde{G} = -\tfrac{1}{2}\Box\left(1 - 6\textstyle\sum_0^\infty c_i \Box^{i+1}\right)h = -\tfrac{1}{2}\Box\Gamma(\Box)h$$

- Potential for h

$$\widetilde{G} \sim -m\delta(\vec{r}) \Rightarrow h(r) \sim \frac{1}{r}\int_{-\infty}^{\infty} dp \frac{p}{p^2\Gamma(-p^2)} e^{ipr} \equiv \frac{G_N(r)}{r}$$

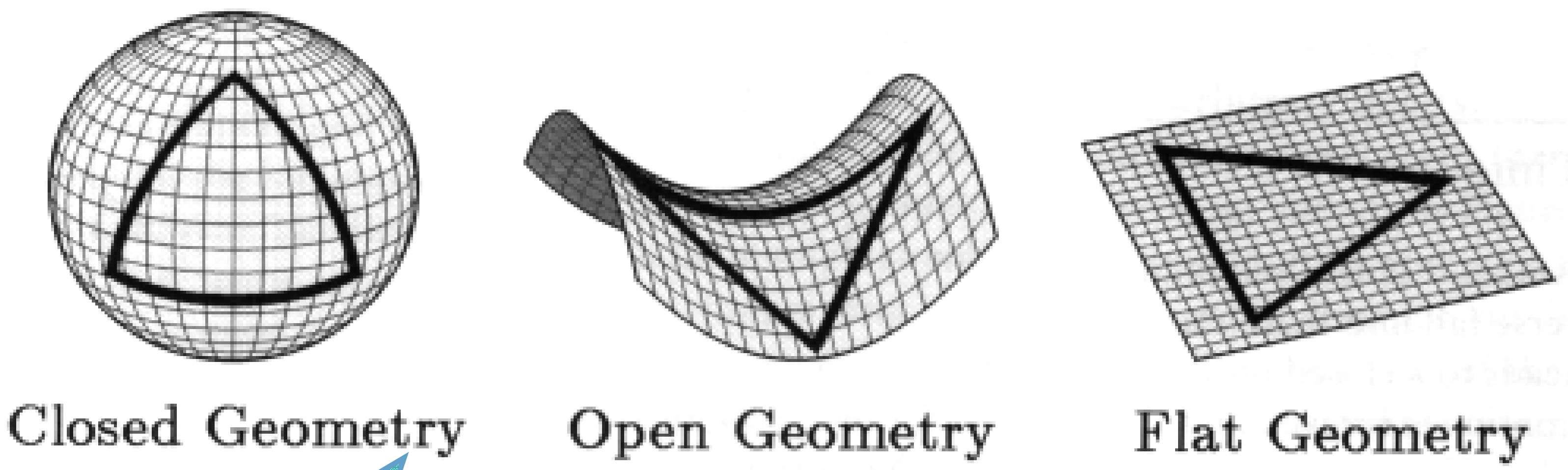
- AF: falls off exponentially $\Rightarrow G_N(r) \to r$
- Newtonian Limit $\Gamma(-p^2) \overset{p\to 0}{\to} 1$ $\Rightarrow G_N(r) \to const$.

- Example: $\Gamma(\Box) = e^{-\Box} \Rightarrow h(r) \sim \frac{\mathtt{erf}(r)}{r}$

Big Bang Singularity

- In GR at $t = 0$ we encounter a singularity

  $R, \Box R, \rho \to \infty$



~~SEC~~

$\rho + 3p < 0$

~~SEC,DEC,WEC~~

$\rho < 0$

Non-singular Bounce

- Ansatz: Find a(t) such that $\Box R \sim R$
  (…)R(t)+(…)R^2(t) ~ matter sources
  Reduces to solving algebraic equation

- Hyperbolic Bounce $a(t) = \cosh(\lambda t)$ works!
- Evolution

$$\tilde{G}_{00} = T_{00} = \frac{1}{3}(\Lambda + \rho_{rad})$$

$$\omega = \frac{1}{3} \Rightarrow \rho_{rad} \sim \frac{1}{a^4} \quad \& \quad a \sim t^{1/2}$$

- Solutions exist for asymptotically and ghost free theories: