CFS INSTABILITY

(Chandrasekhar 1970; Friedman & Schutz 1978)

- **NORMAL MODE** $\delta v \propto e^{-i(\omega t - m\phi)}$
- **INERTIAL FRAME FREQUENCY ω >0**
- **ROTATING FRAME FREQUENCY**

$$\sigma = \omega - m\Omega \qquad (\sigma > 0 \vee \sigma < 0)$$

- **ROTATING FRAME *ENERGY*** $E_{ROT} = E - \Omega J_z$
- ***RADIATION*** $\dot{J}_z = m\dot{E}/\omega \;\rightarrow\; \dot{E}_{ROT} = \sigma \dot{E}/\omega$
- ***RETROGRADE*** ⇨ ***INSTABILITY***

R-MODES

(Andersson 1998; Bildsten 1998;
Friedman & Morsink 1998)

Restoring force = CORIOLIS

$$\frac{\omega}{\Omega} = \frac{(L-1)(L+2)}{L+1} \qquad \frac{\sigma}{\Omega} = -\frac{2}{L+1}$$

★ **SUBSET OF INERTIAL MODES** ★

$$-2\Omega \leq \sigma \leq +2\Omega$$

BASICS I: GROWTH & DAMPING

- Growth rate for R mode:

$$\gamma_{GR} = \frac{8\mathrm{GM}\langle r^4\rangle}{225\mathrm{c}^7}\left(\frac{4\Omega}{3}\right)^6 \approx \frac{M_{1.4}R_6^4\nu_{kHz}^6}{46 \quad sec} \qquad [N=1 \quad polytrope]$$

- Weak dependence on structure

$$\langle r^4\rangle = (0.06-0.09)\left(\frac{M}{\rho_{1/2}}\right)^{4/3} \qquad [N\leq 3.0 \quad polytropes]$$

BASICS I: GROWTH & DAMPING

- Shear boundary layer (low T) [m=2, N=1]

$$\gamma_{BL} = \frac{\sqrt{4/3}\,\pi S^2 \rho_b r_b{}^6 \sqrt{\Omega \eta_{b,visc}}}{M\langle r^4\rangle} \approx \frac{S^2 \rho_{b,14} R_6^2 \sqrt{\nu_{kHz}(T_8^2 \eta_{b,visc})_4}}{200\, T_8 M_{1.4}\ \ sec}$$

Crust-core boundary; shear modulus

- Other modes:

$$\gamma_{BL} \sim \frac{\rho_b r_b^2 S_{mode}^2 v_{mode}^2(r_b)\sqrt{\Omega \eta_{b,visc}}}{E_{mode}}$$

BASICS I: GROWTH & DAMPING

- Bulk viscosity: (High T)

$$\gamma_{BULK} = \frac{\omega}{2E_{mode}} \int dV\, xP(\nabla\cdot\boldsymbol{\xi})^2 F\left(\Gamma_{relax}/\omega\right)$$

$$F(Z) = \frac{2Z}{1+Z^2} \leq 1 \quad \sim 2Z \quad [Z \ll 1] \quad \sim 2/Z \quad [Z \gg 1]$$

- Suppressed for R mode(s) $\left(|\nabla\cdot\boldsymbol{\xi}| \sim \Omega^2/G\rho\right)$
- Depends on composition, EOS

BASICS II: DYNAMICS

- **RESONANCES :** $\delta\omega = |\omega_R - \omega_2 - \omega_3| \ll \Omega$
- ***SELECTION RULES & COUPLING***
- **Parametric instability threshold *PIT*** (3 mode)
- ***Steady state amplitudes*** (3 mode)
- ***LOWEST PIT*** (3 or many mode)

$$|c_R|^2_{PIT} = \frac{\gamma_2\gamma_3}{4\kappa^2\omega_2\omega_3}\left[1+\frac{(\delta\omega)^2}{(\gamma_2+\gamma_3)^2}\right] \sim \frac{\gamma_2\gamma_3}{\kappa^2\omega_2\omega_3} \sim \frac{(\delta\omega)^2}{\kappa^2\omega_2\omega_3}$$

BASICS III: HEATING & COOLING

- **Heating by modes:**

$$H_{modes} \approx 2\mathrm{MR}^2 \Omega^2 \gamma_{GR} |c_R|^2 = \frac{MR^2 \Omega^2 \gamma_{GR} \gamma_2 \gamma_3}{2\kappa^2 \omega_2 \omega_3} (1+\Delta^2)$$

- **Neutrino cooling:** $L_\nu(T) \qquad [dL_\nu(T)/dT > 0]$

- **Heating by accretion:** $H_{accretion} = \epsilon \dot{M} c^2$

- **Evolve toward *BALANCE***

$$L_\nu(T) = H_{modes} + H_{accretion} \rightarrow \Omega(T)$$

GRAVITATIONAL RADIATION

- **Spindown: gravitational radiation of R mode**

$$h_{rms}^2 t_{spindown} = \frac{3GI}{2c^3 d^2} \approx \frac{(1.1\times10^{-25})^2 I_{45}}{[d(Mpc)]^2} \quad years$$

- **Best case: spindown along heating=cooling**

$$t_{spindown} = \frac{I\Omega^2}{3[L_\nu(T) - \epsilon \dot{M} c^2]} \approx \frac{I\Omega^2}{3L_\nu(T)}$$

- **Moment of inertia, cooling** *(damping, T)*

VIRTUES & PITFALLS

- **LMXBs:** $t_{spindown} \sim (1-100)\times 10^3\ years$

  $\Rightarrow$ **long lifetime, weaker source**

- **NEWBORNS:** $t_{spindown} \sim 1-10\ \ years \quad [Type\ \ I]$

  $\Rightarrow$ **shorter lifetime, stronger source**

- **Phase slippage:**

$$\Delta\phi = \nu t_{obs}^2 / 2 t_{spindown} \approx 1.2\times 10^5 \nu_{kHz} [t_{obs}(days)]^2 / [t_{spindown}(years)]$$

- **Nonlinear effects: limit cycles, chaos**