Nuclear Matter Equation of State

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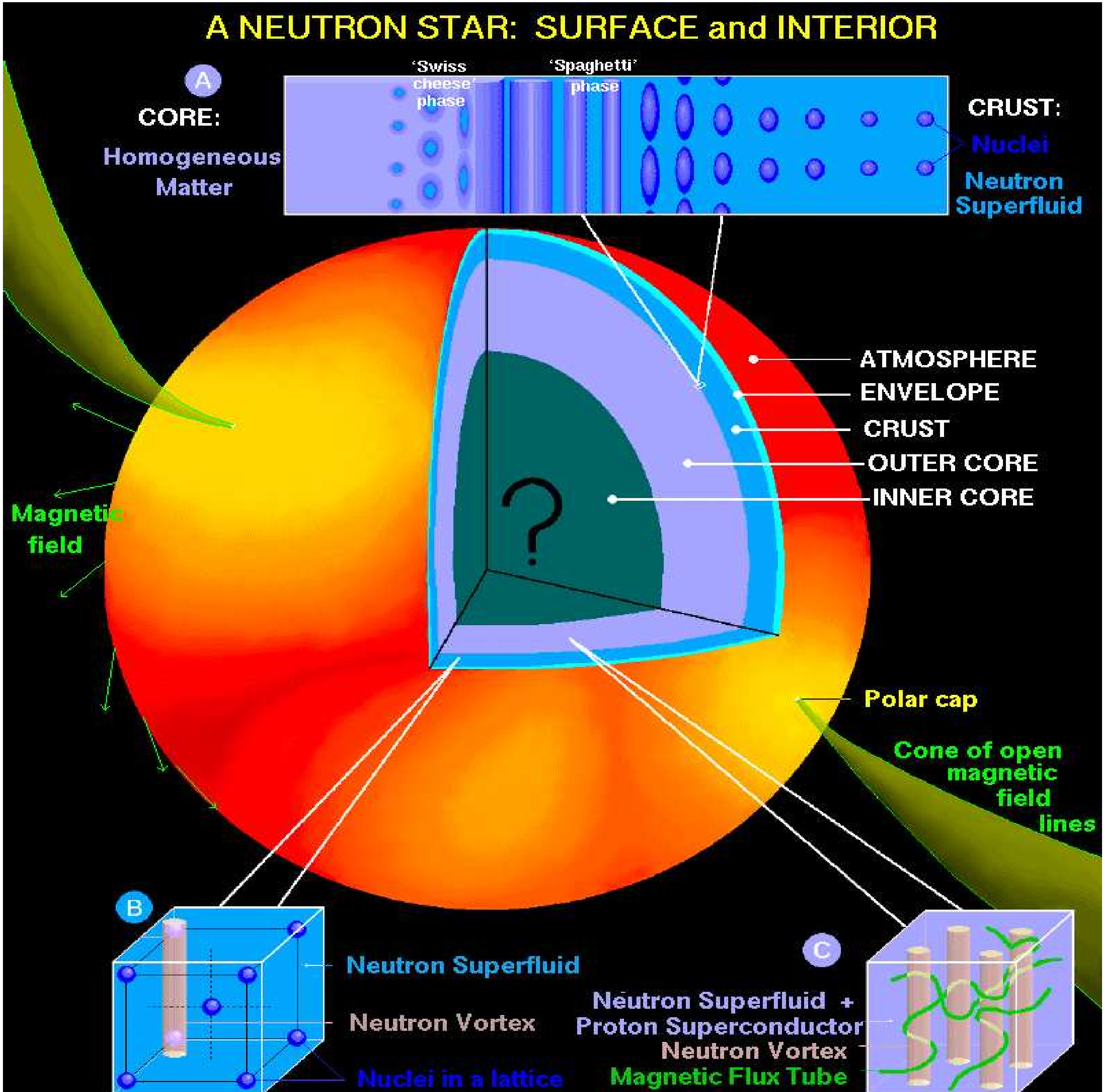
Neutron Stars and the Equation of State

- Extreme Properties
- Pulsar Constraints – Rotation and Mass
- Pressure–Radius Correlation and the Nuclear Symmetry Energy
- Observational Mass and Radius Constraints
- Gravitational Radiation and Tidal Love Numbers



Credit: Dany Page, UNAM

Relevant Observations

- Maximum and Minimum Mass (Binary Pulsars)
- Minimum Rotational Period
- Radiation Radii or Redshifts from X-ray Thermal Emission
- Crustal Cooling Timescale from X-ray Transients
- X-ray Bursts from Accreting Neutron Stars
- Seismology from Giant Flares from Magnetars
- Pulsar Glitches
- Long-Term Neutron Star Cooling (URCA or not)
- Moments of Inertia from Spin-Orbit Coupling
- Neutrinos from Proto-Neutron Stars (Binding Energies, Neutrino Opacities, Radii)
- Redshifts from Pulse Shape Modulation
- Gravitational Radiation from Mergers (Masses from Inspiral, Radii from Tides)

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations of relativistic hydrostatic equilibrium:

$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m+4\pi pr^3)(\epsilon+p)}{r(r-2Gm/c^2)}$$
$$\frac{dmc^2}{dr} = 4\pi\epsilon r^2$$

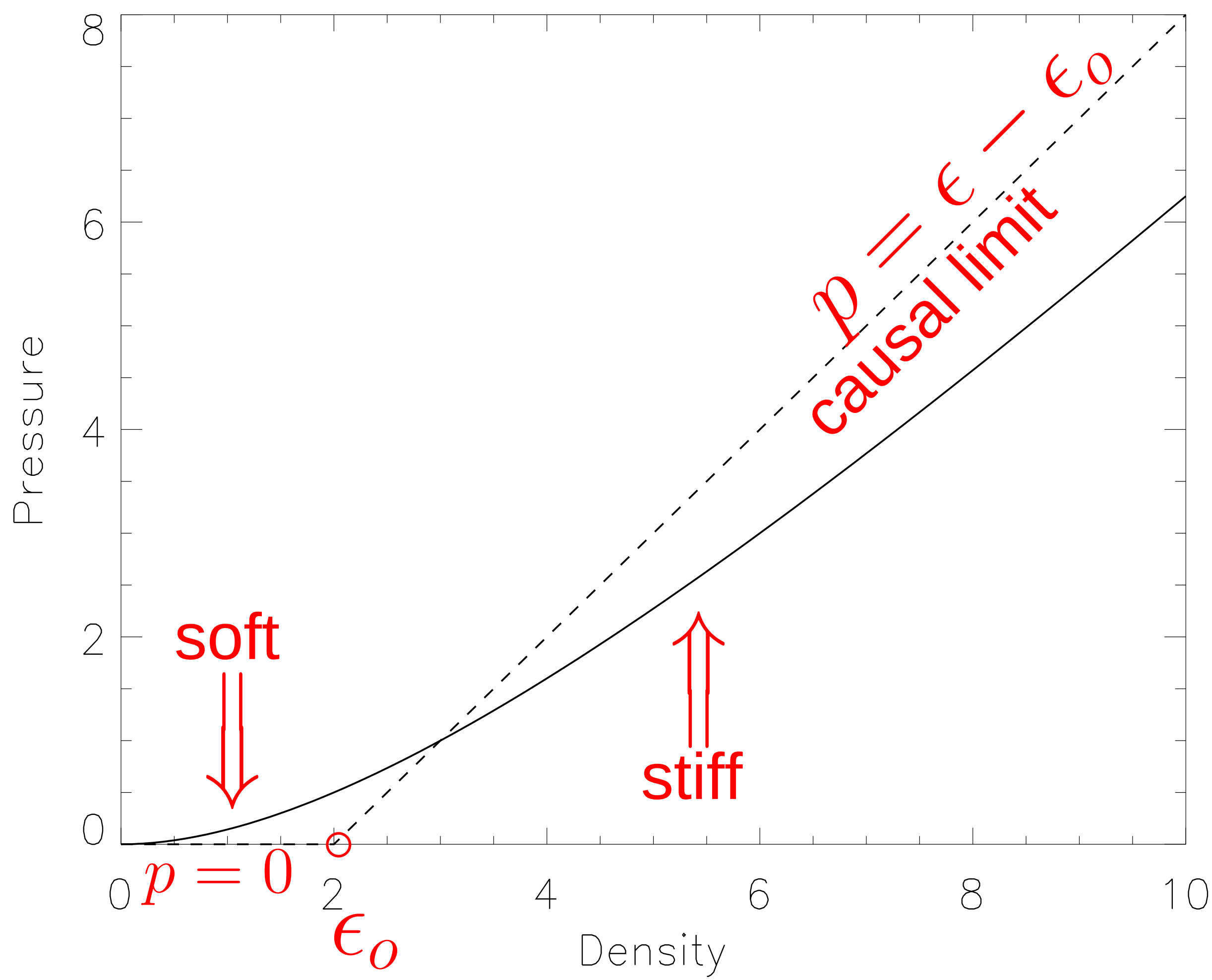
p is pressure, ϵ is mass-energy density

Useful analytic solutions exist:

- Uniform density $\epsilon =$ constant
- Tolman VII $\epsilon = \epsilon_c[1-(r/R)^2]$
- Buchdahl $\epsilon = \sqrt{pp_*} - 5p$
- Tolman IV $\nu(r) = \nu(0) + N\ln\left[1+(r/a)^2\right]$

Extreme Properties of Neutron Stars

- The most compact configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff".



ϵ_0 is the only EOS parameter

The TOV solutions scale with ϵ_0

Maximum Mass, Minimum Period

Theoretical limits from GR and causality

- $M_{max} = 4.2(\epsilon_s/\epsilon_0)^{1/2}\ \mathrm{M}_\odot$ **Rhoades & Ruffini (1974), Hartle (1978)**
- $R_{min} = 2.9GM/c^2 = 4.3(M/\mathrm{M}_\odot)\ \mathrm{km}$

Lindblom (1984), Glendenning (1992), Koranda, Stergioulas & Friedman (1997)

- $\epsilon_c < 4.5 \times 10^{15}(\mathrm{M}_\odot/M_{largest})^2\ \mathrm{g\ cm}^{-3}$ **Lattimer & Prakash (2005)**
- $P_{min} \simeq (0.74 \pm 0.03)(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}\ \mathrm{ms}$

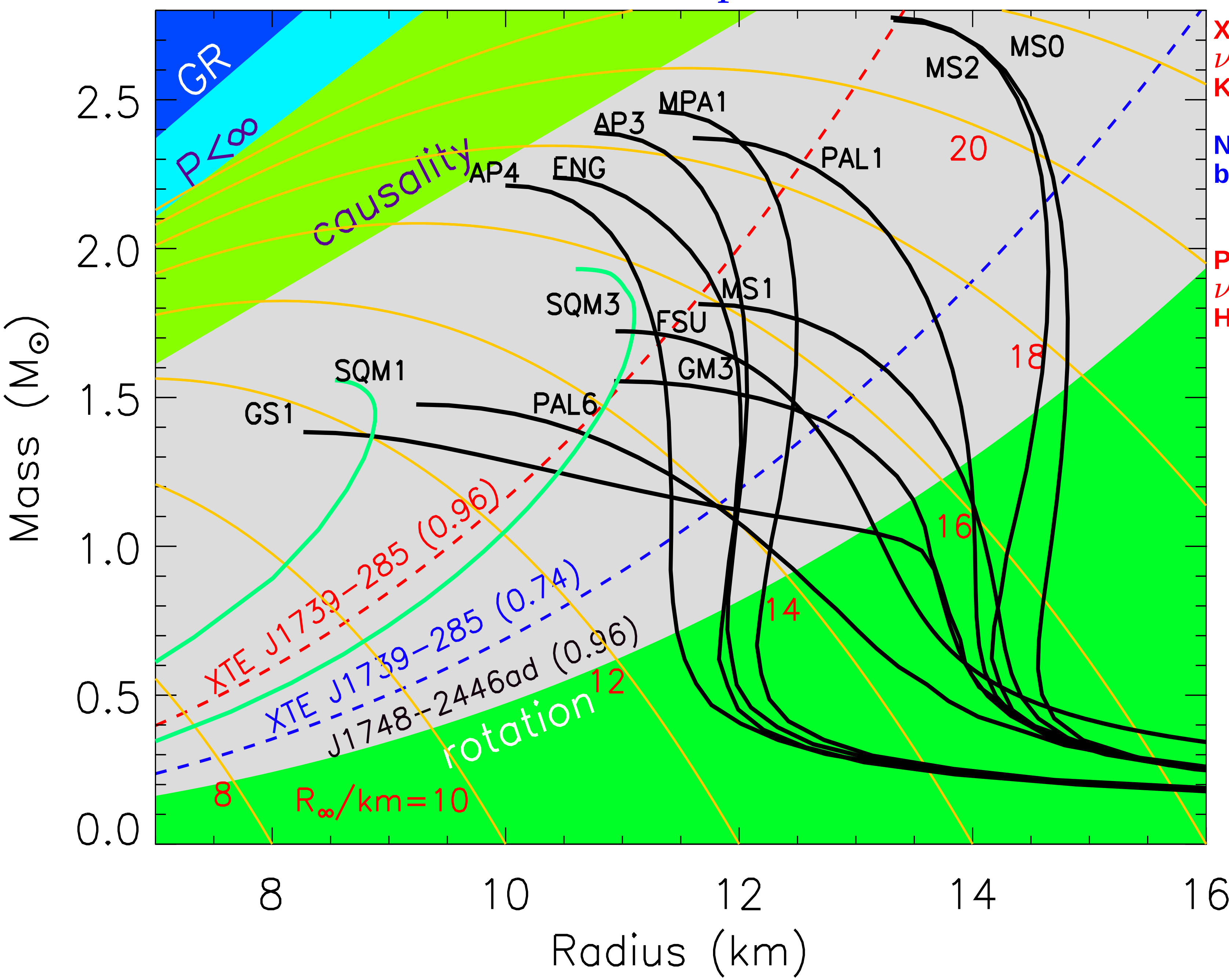
Koranda, Stergioulas & Friedman (1997)

- $P_{min} \simeq 0.96(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}\ \mathrm{ms}$ (empirical)

Lattimer & Prakash (2004)

- $\epsilon_c > 0.91 \times 10^{15}(1\ \mathrm{ms}/P_{min})^2\ \mathrm{g\ cm}^{-3}$ (empirical)
- $cJ/GM^2 \lesssim 0.5$ (empirical, neutron star)

Constraints from Pulsar Spins



XTE J1739-285
$\nu = 1122$ **Hz**
Kaaret et al. 2006

Not confirmed to be a rotation rate

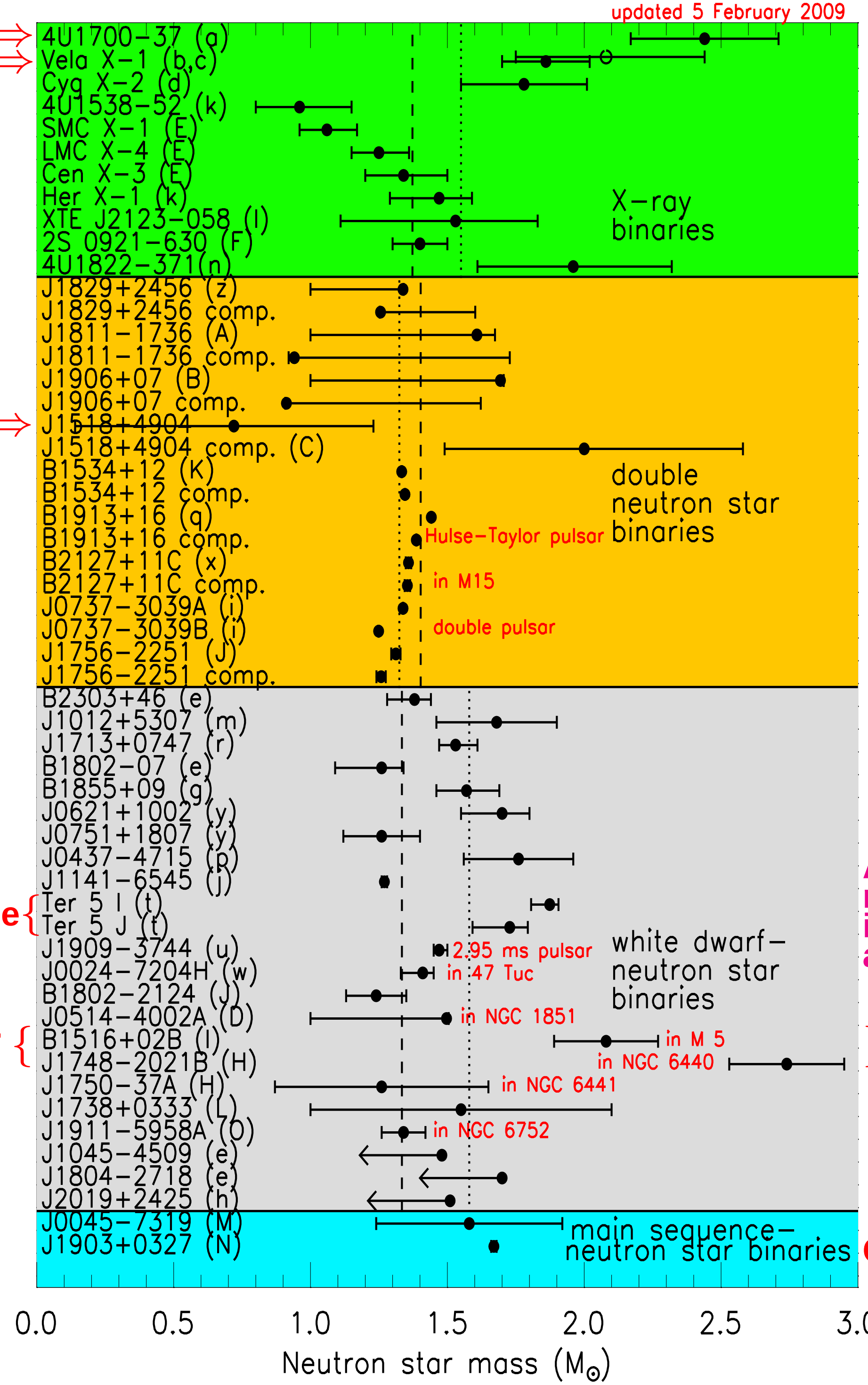
PSR J1748-2446ad
$\nu = 716$ **Hz**
Hessels et al. 2006

Black hole? ⇒
Firm lower mass limit? ⇒

$M < 1.17$ **M**$_\odot$ (95%) ⇒

$M > 1.68$ **M**$_\odot$**, 95% confidence** {

Freire et al. 2007 {

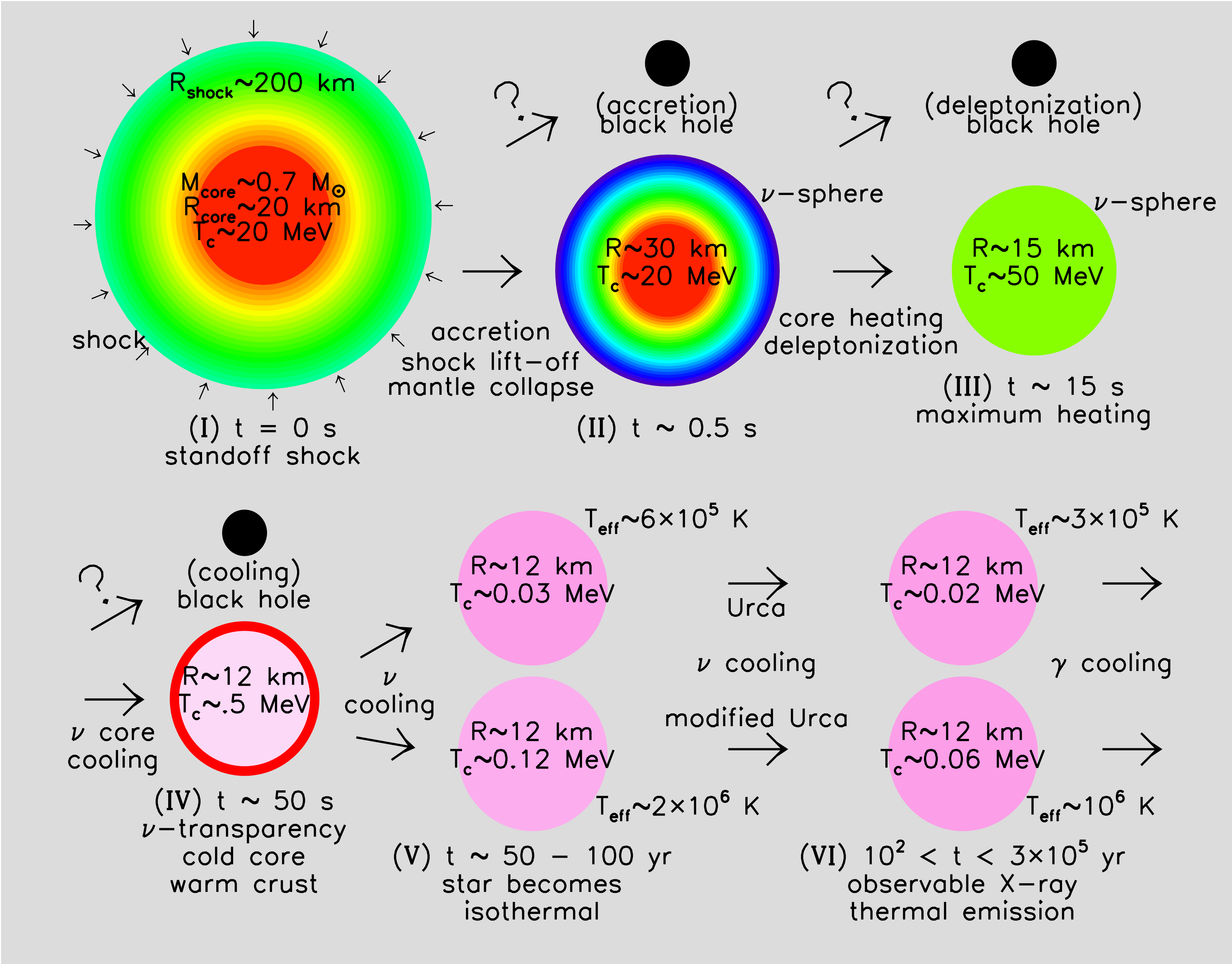


Although simple average mass of w.d. companions is 0.27 M$_\odot$ **larger, weighted average is 0.08 M**$_\odot$ **smaller**

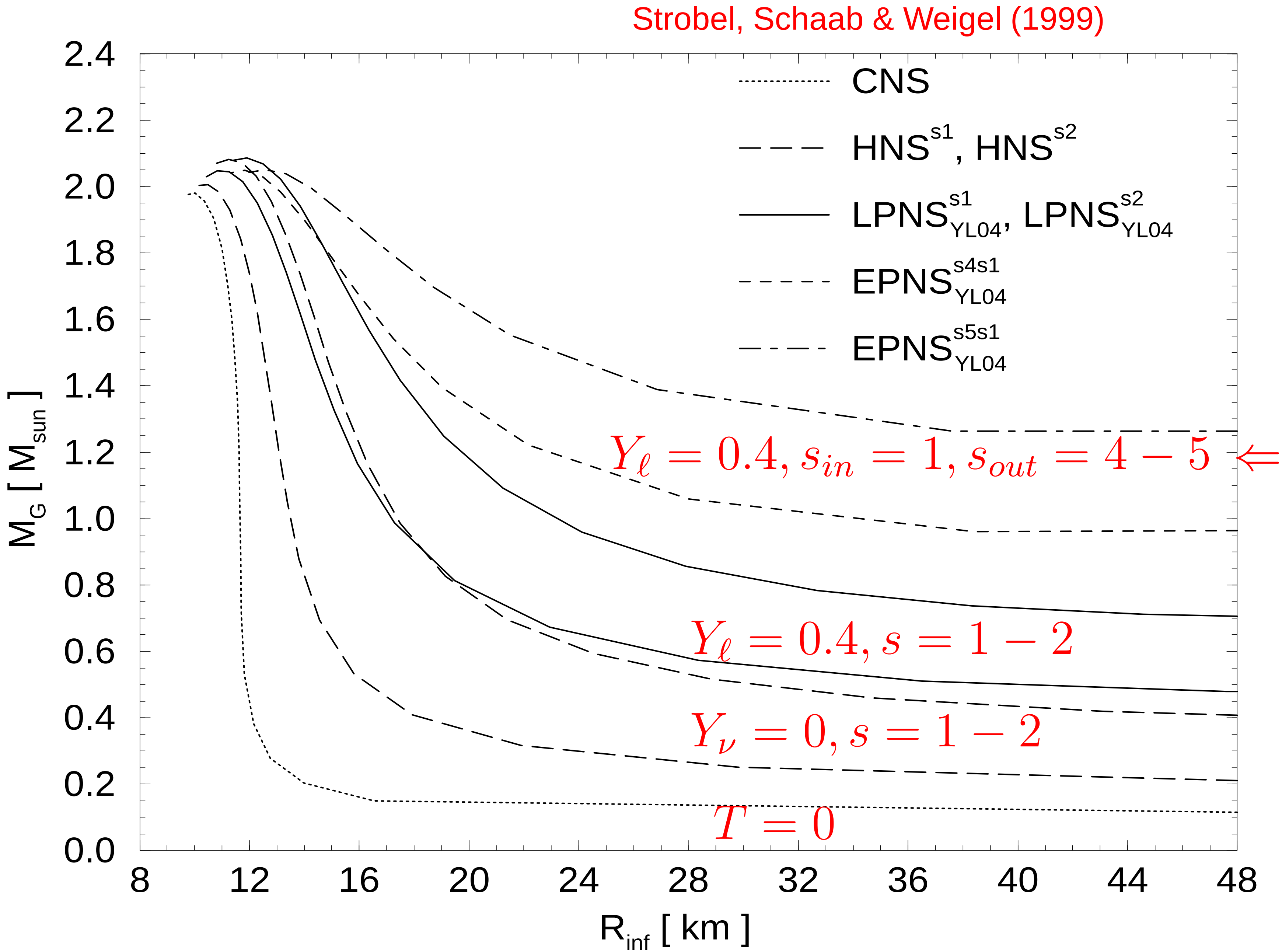
} **w.d. companion? statistics?**

Champion et al. 2008

Proto-Neutron Stars



Effective Minimum Masses



Neutron Star Matter Pressure and the Radius

$$p \simeq K\epsilon^{1+1/n}$$

$$n^{-1} = d\ln p/d\ln\epsilon - 1 \sim 1$$

$$R \propto K^{n/(3-n)} M^{(1-n)/(3-n)}$$

$$R \propto p_*^{1/2}\epsilon_*^{-1} M^0$$

$$(1 < \epsilon_*/\epsilon_0 < 2)$$

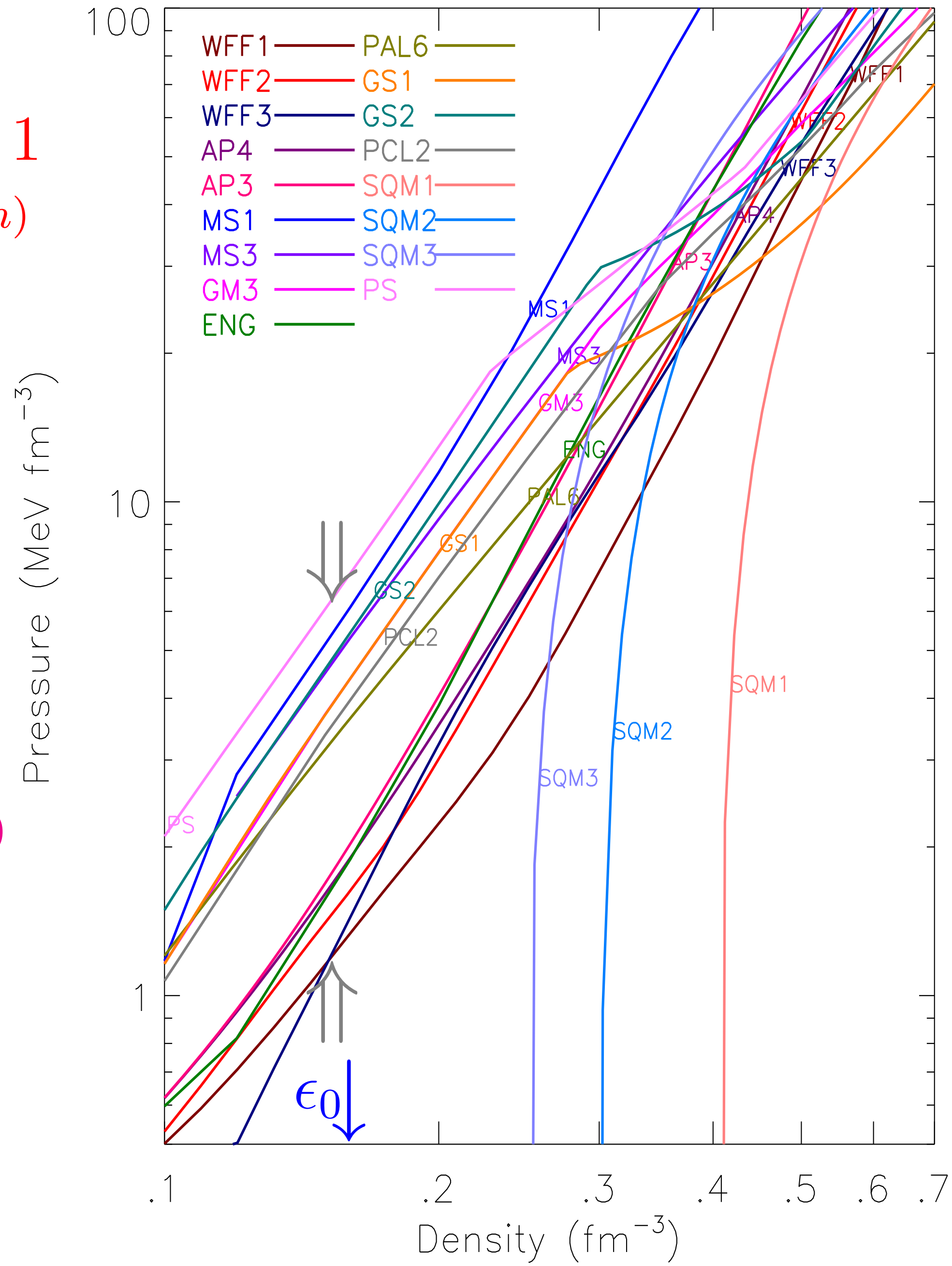
Wide variation:

$$1.2 < \frac{p(\epsilon_0)}{\mathrm{MeV\ fm}^{-3}} < 7$$

GR phenomenological result (Lattimer & Prakash 2001)

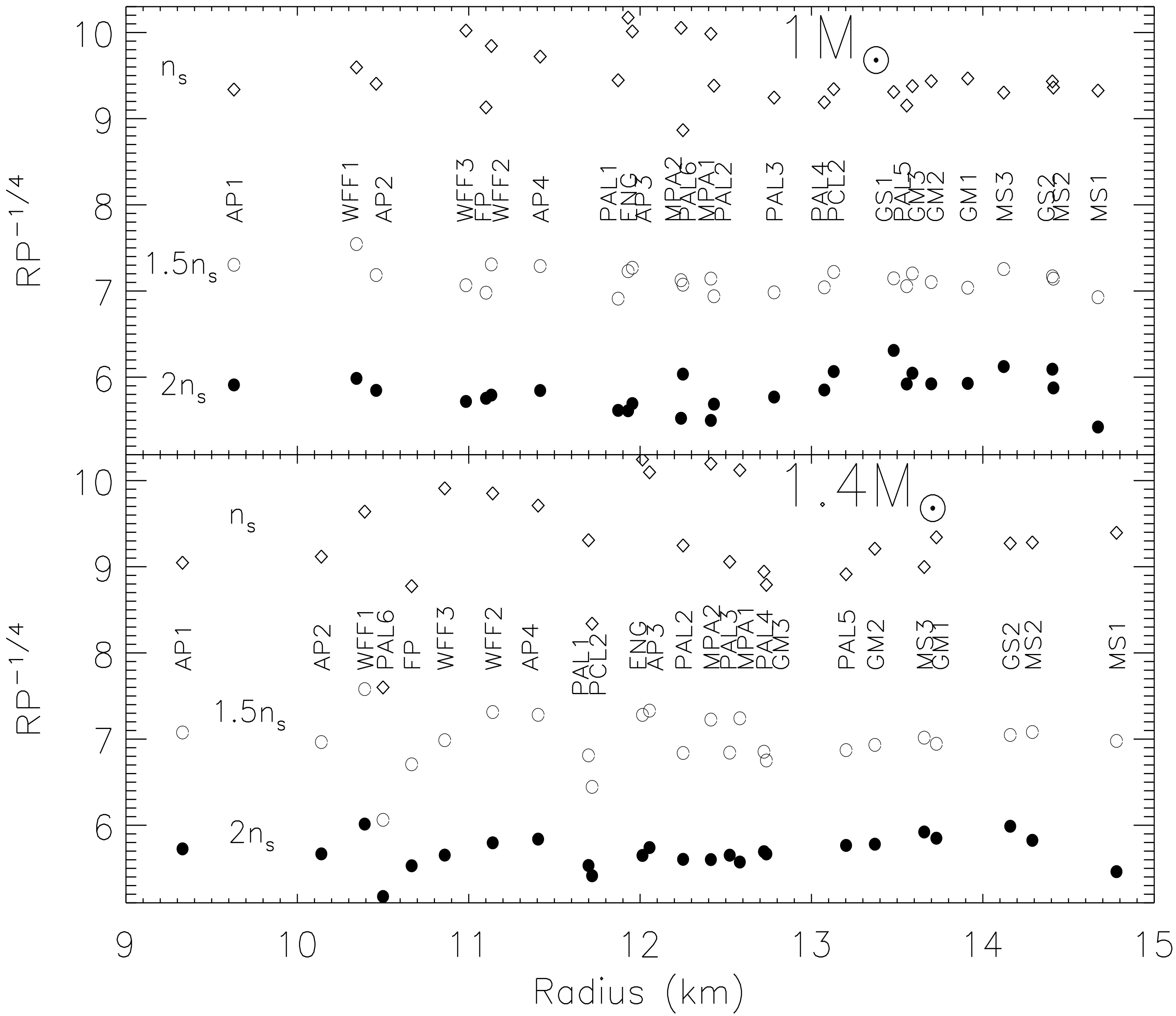
$$R \propto p_*^{1/4}\epsilon_*^{-1/2}$$

$$p_* = n^2\frac{dE_{sym}}{dn} = \frac{n^2 L}{3n_s}$$



The Radius – Pressure Correlation

$R \propto p^{1/4}$



Lattimer & Prakash (2001)

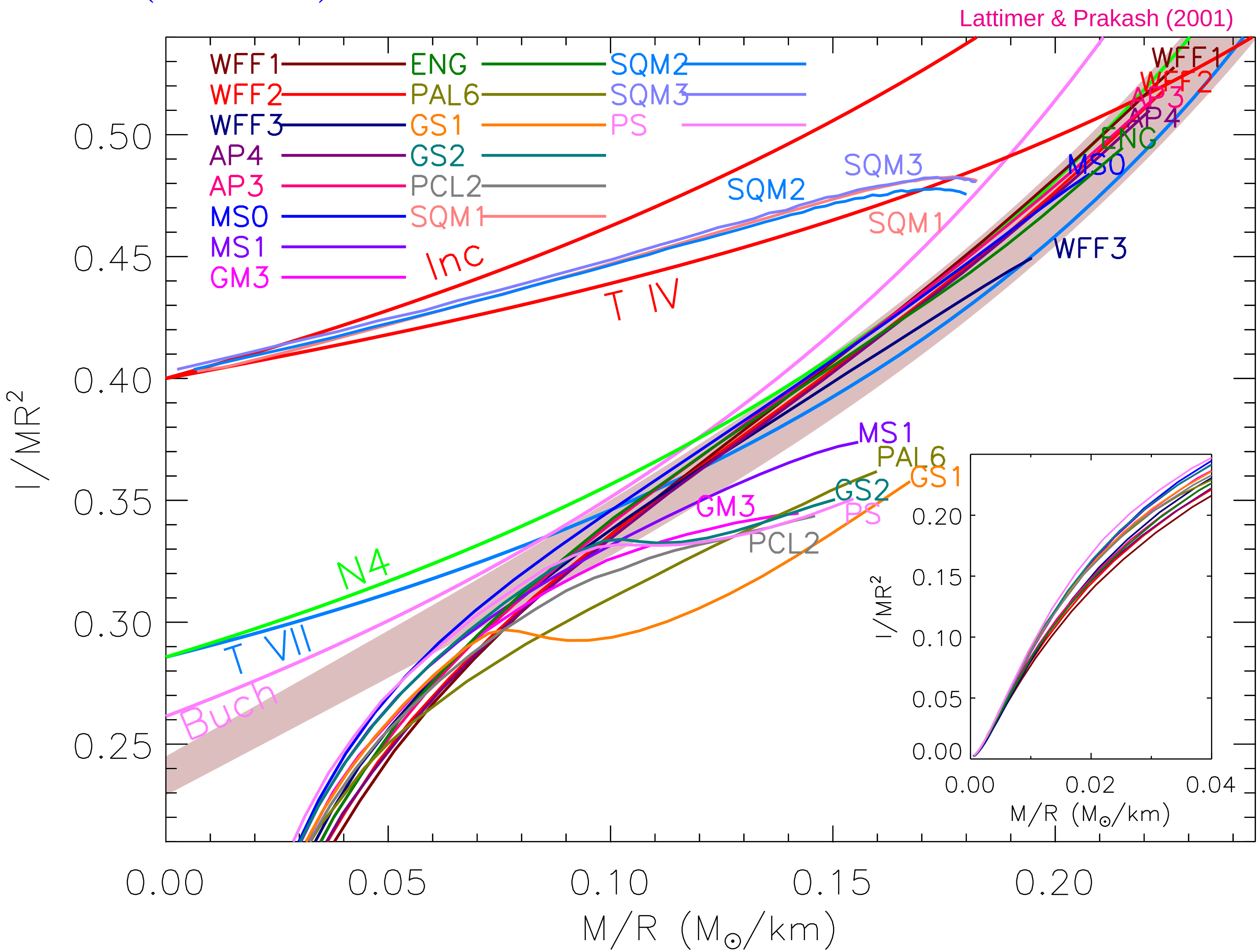
Potentially Observable Quantities

- Apparent angular diameter from flux and temperature measurements
  $\beta \equiv GM/Rc^2$
  $$\frac{R_\infty}{D} = \frac{R}{D}\frac{1}{\sqrt{1-2\beta}} = \sqrt{\frac{F_\infty}{\sigma}}\frac{1}{f_\infty^2 T_\infty^2}$$
- Redshift
  $$z = (1-2\beta)^{-1/2} - 1$$
- Eddington flux
  $$F_{EDD} = \frac{GMc}{\kappa c^2 D^2}(1-2\beta)^{1/2}$$
- Crust thickness ($\ln \mathcal{H} = (2/m_b c^2)\int_0^{p_t}(dp/n)$)
  $$\frac{\Delta}{R} \equiv \frac{R-R_t}{R} = \frac{(\mathcal{H}-1)(1-2\beta)}{\mathcal{H}+2\beta-1} \simeq (\mathcal{H}-1)\left(\frac{1}{2\beta}-1\right).$$
- Moment of Inertia
  $$I \simeq (0.237 \pm 0.008) MR^2 (1 + 2.84\beta + 18.9\beta^4)\ \mathrm{M}_\odot\ \mathrm{km}^2$$
- Crustal Moment of Inertia
  $$\frac{\Delta I}{I} \simeq \frac{8\pi}{3}\frac{R^6 p_t}{IMc^2}$$
- Binding Energy
  $$\mathrm{B.E.} = \frac{N-M}{M} \simeq (0.60 \pm 0.05)\frac{\beta}{1-\beta/2}$$
- Tidal Love Number
  $$Q_{ij} = -\lambda E_{ij}, \qquad \lambda = (2/3G)k_2 R^5$$

Moment of Inertia

- Spin-orbit coupling of same magnitude as post-post-Newtonian effects (Barker & O'Connell 1975, Damour & Schaeffer 1988)
- Precession alters inclination angle and periastron advance
- More EOS sensitive than R: $I \propto MR^2$
- Requires extremely relativistic system to extract
- Double pulsar PSR J0737-3037 is a marginal candidate
- Even more relativistic systems should be found, based on dimness and nearness of PSR J0737-3037

$I(M,R)$



$$I \simeq (0.237 \pm 0.008) M R^2 \left[1 + 4.2 \frac{M \text{ km}}{R \text{ M}_\odot} + 90 \left(\frac{M \text{ km}}{R \text{ M}_\odot} \right)^4 \right]$$

Moments of Inertia and Precession

- Spin-orbit coupling: $\dot{\vec{S}}_A = -\dot{\vec{L}} = \frac{G(4M_A+3M_B)}{2M_A a^3 c^2} \vec{L} \times \vec{S}_A$

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$$\delta_i = \frac{|\vec{S_A}|}{|\vec{L}|}\sin\theta \simeq \frac{I_A(M_A + M_B)}{a^2 M_A M_B}\frac{P}{P_A}\sin\theta \simeq (3.6 - 7.2)\sin\theta \times 10^{-5}$$

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$$\frac{2\pi I_A}{P_A}\left(\frac{2+3M_B/M_A}{3M_A^2+3M_B^2+2M_AM_B}\right)\sqrt{\frac{M_A+M_B}{Ga}}\cos\theta \simeq (2.2-4.3)\times 10^{-4}\cos\theta$$

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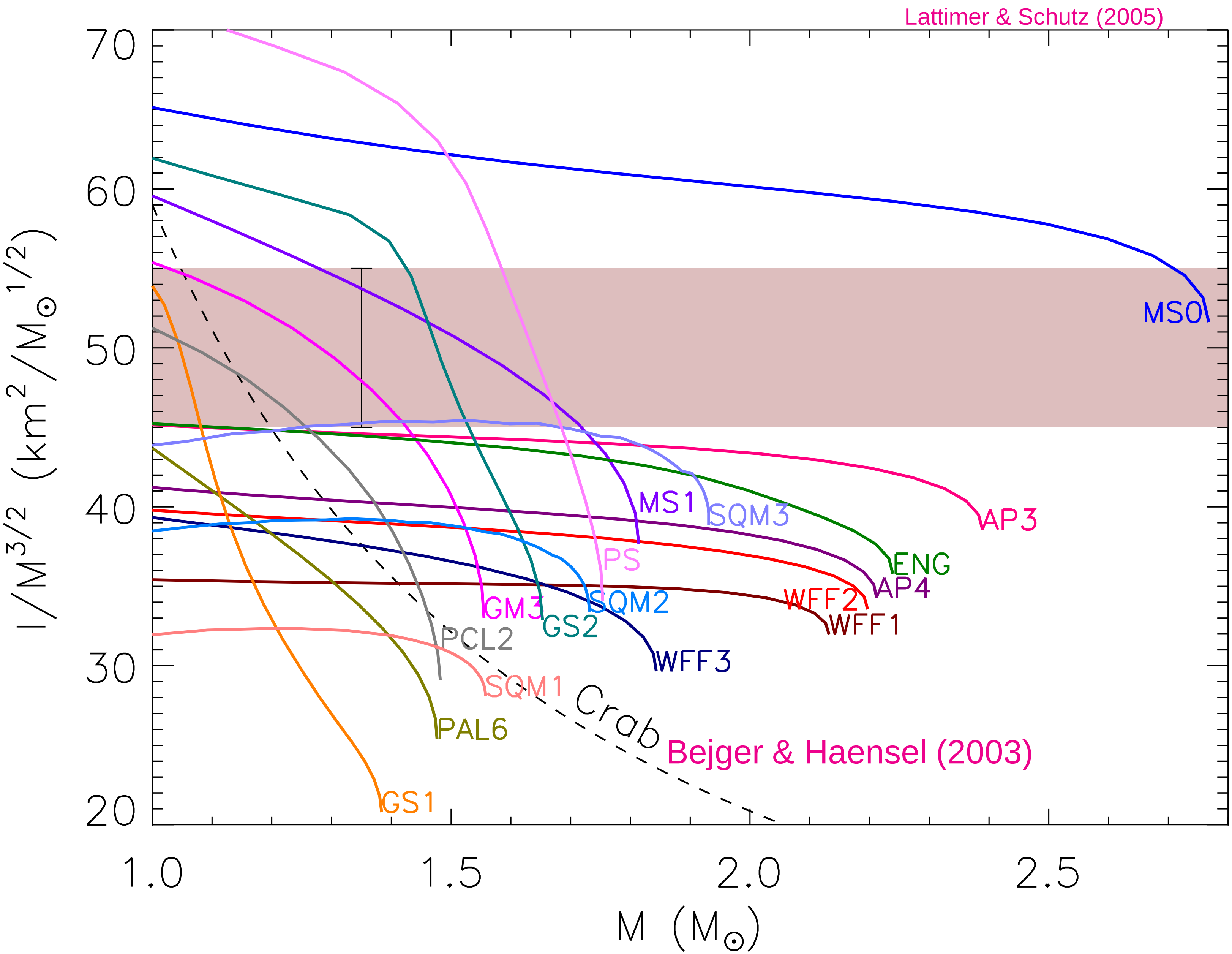
Comparison of Binary Pulsars

References	PSR 0707-3039 a, b, c	PSR 1913+16 d	PSR 1534+12 e, f
a/c **(s)**	**2.93**	**6.38**	**7.62**
P **(h)**	**2.45**	**7.75**	**10.1**
e	**0.088**	**0.617**	**0.274**
M_A **($M_\odot$)**	1.337 ± 0.005	1.4414 ± 0.0002	1.333 ± 0.001
M_B **($M_\odot$)**	1.250 ± 0.005	1.3867 ± 0.0002	1.345 ± 0.001
T_{GW} **(M yr)**	**85**	**245**	**2250**
i	$87.9 \pm 0.6^\circ$	47.2°	77.2°
P_A **(ms)**	**22.7**	**59**	**37.9**
θ_A	$13^\circ \pm 10^\circ$	21.1°	$25.0^\circ \pm 3.8^\circ$
ϕ_A	$246^\circ \pm 5^\circ$	9.7°	$290^\circ \pm 20^\circ$
P_{pA} **(yr)**	**74.9**	**297.2**	**700**
$\delta t_a/I_{A,80}$ (μ**s**)	0.7 ± 0.6	**11.2**	7.9 ± 1.1
$A_{pA}/(A_{1PN} I_{A,80})$	$3.4^{+0.2}_{-0.1} \times 10^{-5}$	1.0×10^{-5}	$1.15^{+0.04}_{-0.03} \times 10^{-5}$
A_{2PN}/A_{1PN}	5.2×10^{-5}	4.7×10^{-5}	2.3×10^{-5}

a: Lyne et al. (2004); b: Solution 1, Jenet & Ransom (2004); c: Coles et al. (2004)
d: Weisberg & Taylor (2002, 2004); e: Stairs et al. (2002, 2004); f: Bogdanov et al. (2002)

EOS Constraint



Tidal Effects in Mergers

Thesis work of Sergey Postnikov (Ohio University) in collaboration with M. Praksh and JML

- Masses of components of inspiralling neutron stars will be well measured.
- Large finite-size effects like mass exchange and tidal disruption will be visible in the gravitational wave signal toward the end of inspiral
- However, the gravitational wave signal is very complex during this period
- Tidal effects are potentially measurable during the earlier part of the evolution when the waveform is relatively clean; it is a cumulative effect
- The effect depends on the induced quadrupole moment Q_{ij}, which is proportional to the applied tidal field E_{ij}
- In early evolution, tidal effects form a very small correction in which the accumulated phase shift is characterized by the weighted average Q_{ij} for the two stars.
- The proportionality constant depends on the stellar radius and the internal structure: $\lambda = (2/3G)k_2 R^5$
- The tidal Love number k_2 depends on the equation of state and compactness β
- Pronounced differences in k_2 exist between neutron stars and strange quark stars
- The extreme dependence on R offers a possibility of a constraint

Computation of Tidal Love Numbers

We follow Thorne & Campolattaro (1967 ApJ 149, 591) and Hinderer (2008 ApJ 677, 1216):

$$\begin{aligned}
k_2(\beta, y_R) &= \frac{8}{5}\beta^5(1-2\beta)^2\left[2-y_R+2\beta(y_R-1)\right]\times \\
&\times \left\{2\beta\left(6-3y_R+3\beta(5y_R-8)+2\beta^2\left[13-11y_R+\beta(3y_R-2)+2\beta^2(1+y_R)\right]\right.\right. \\
&+ \left. 3(1-2\beta)^2\left[2-y_R+2\beta(y_R-1)\right]\ln(1-2\beta)\right\}^{-1}.
\end{aligned}$$

$$y_R = \left[rH'(r)/H(r)\right]_{r=R}$$

$$H''(r)+H'(r)\left[\frac{2}{r}+e^{\lambda(r)}\left(\frac{2m(r)}{r^2}+4\pi r[p(r)-\rho(r)]\right)\right]+H(r)Q(r)=0,$$

$$Q(r)=4\pi e^{\lambda(r)}\left(5\rho(r)+9p(r)+\frac{\rho(r)+p(r)}{c_s^2(r)}\right)-6\frac{e^{\lambda(r)}}{r^2}-\left(\nu'(r)\right)^2 .$$

Can be simplified:

$$ry'(r)+y(r)^2+y(r)e^{\lambda(r)}\left(1+4\pi r^2[p(r)-\rho(r)]\right)+r^2Q(r)=0\,,$$

$$y(0)=2$$

Newtonian Limit

$$p << \rho, \qquad \rho r^2 << 1$$

$$\begin{aligned} k_2(y_R) &= \frac{1}{2}\frac{2-y_R}{3+y_R}\,, \\ r y'(r) &+ y(r)^2 + y(r) - 6 + 4\pi r^2 \frac{\rho(r)}{c_s^2(r)} = 0\,. \end{aligned}$$

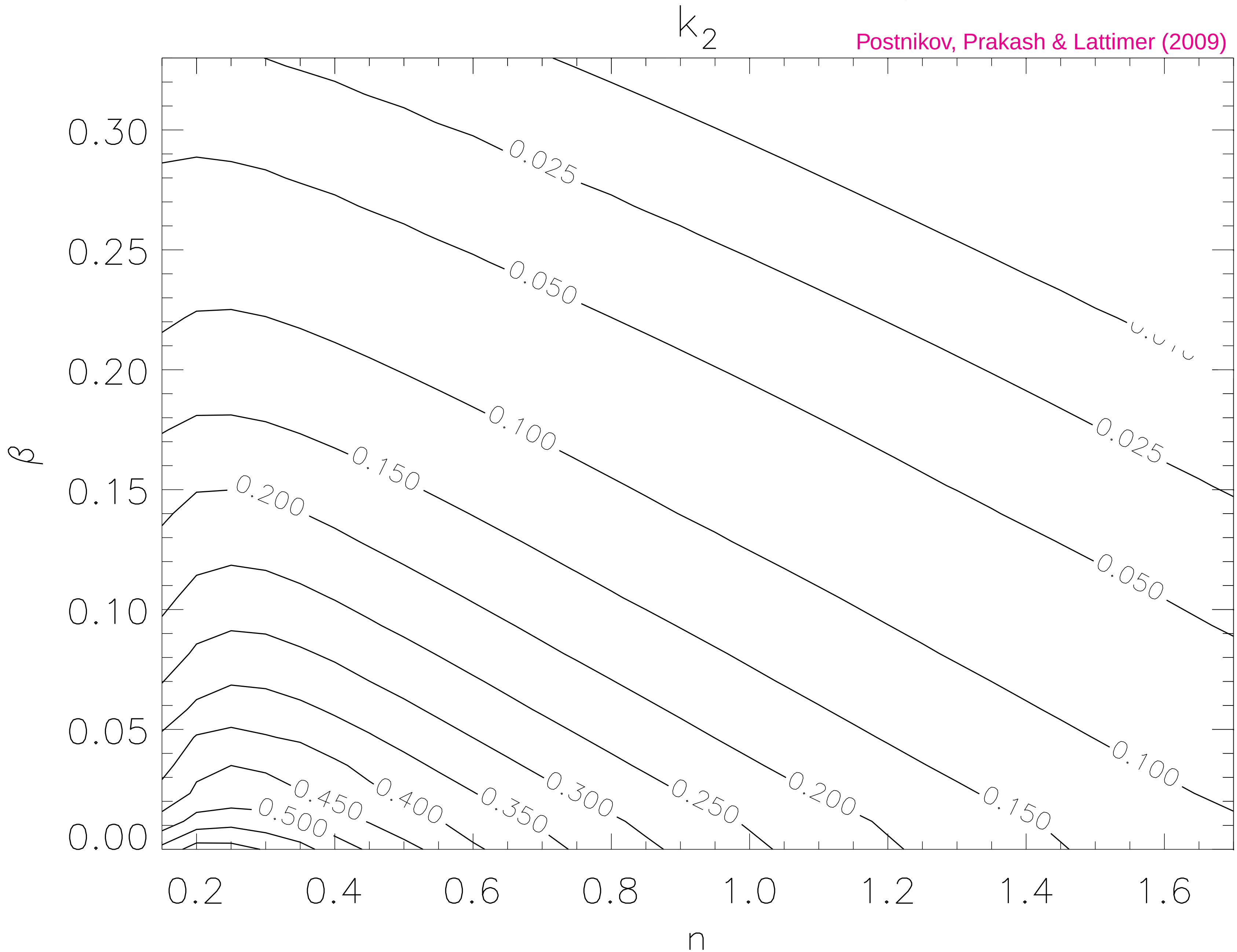
Analytic polytropic cases:

$$n = 0: \quad y(r) = 2, \qquad k_2 = 0$$

But Damour & Nagar (archiv:0906.0096) claim that the density discontinuity at the surface changes this such that $y_R = -1$ and $k_2 = 3/4$.

$$n = 1: \qquad y(r) = \frac{\pi r}{R}\frac{J_{3/2}(\pi r/R)}{J_{5/2}(\pi r/R)} - 3, \qquad y_R = \frac{\pi^2 - 9}{3}, \qquad k_2 = \frac{15 - \pi^2}{2\pi^2}$$

Tidal Love Numbers of Polytropes



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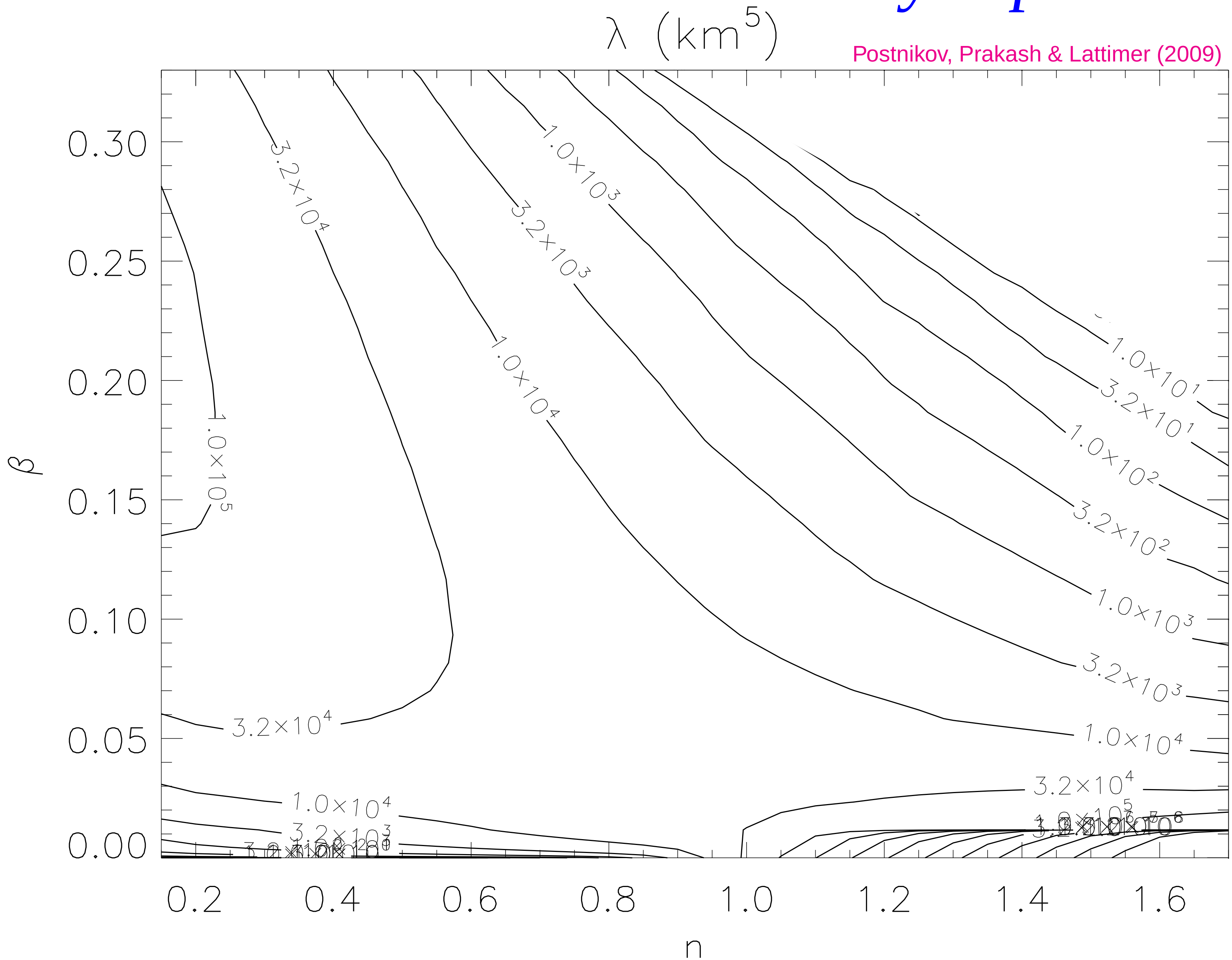
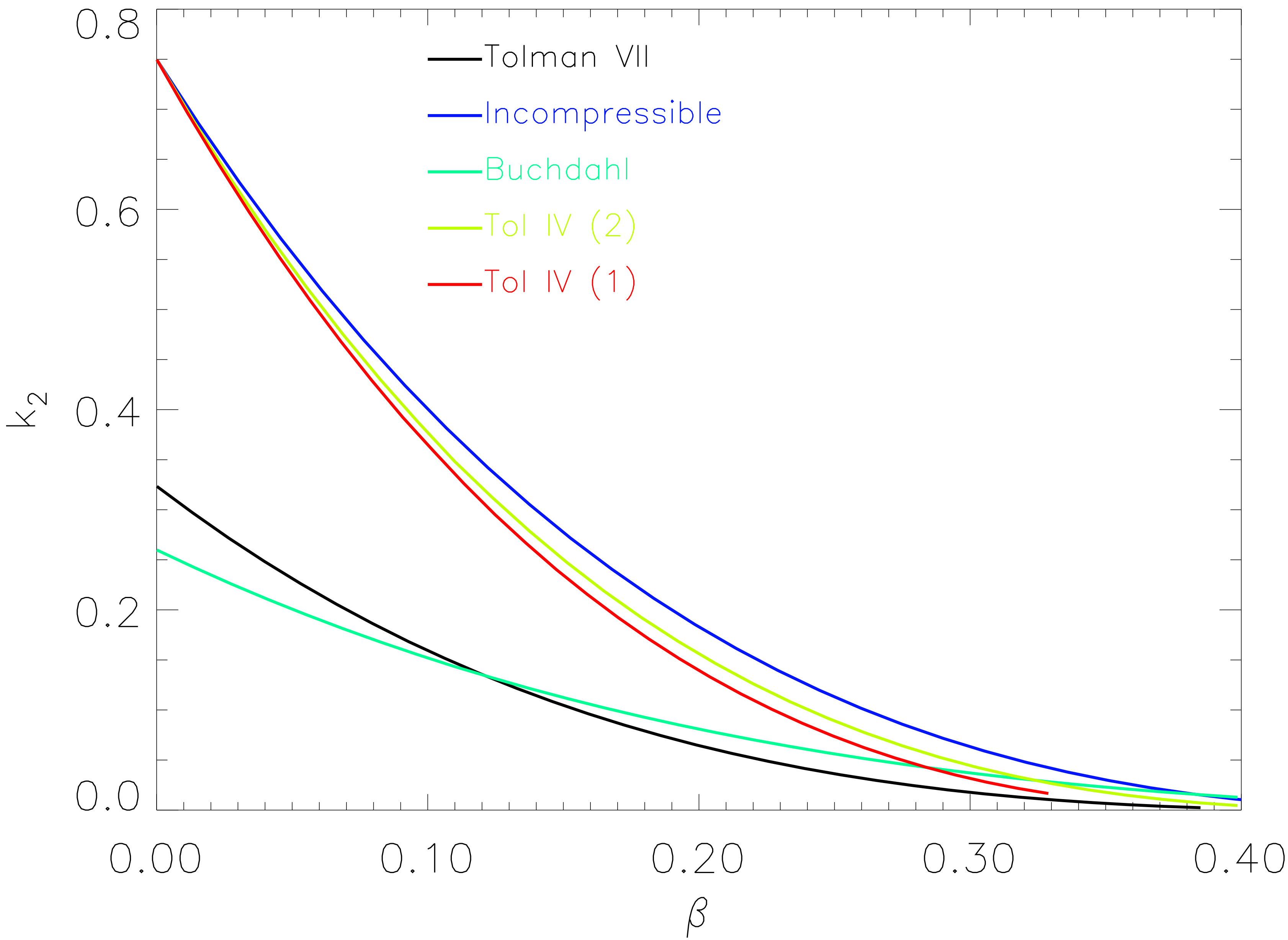
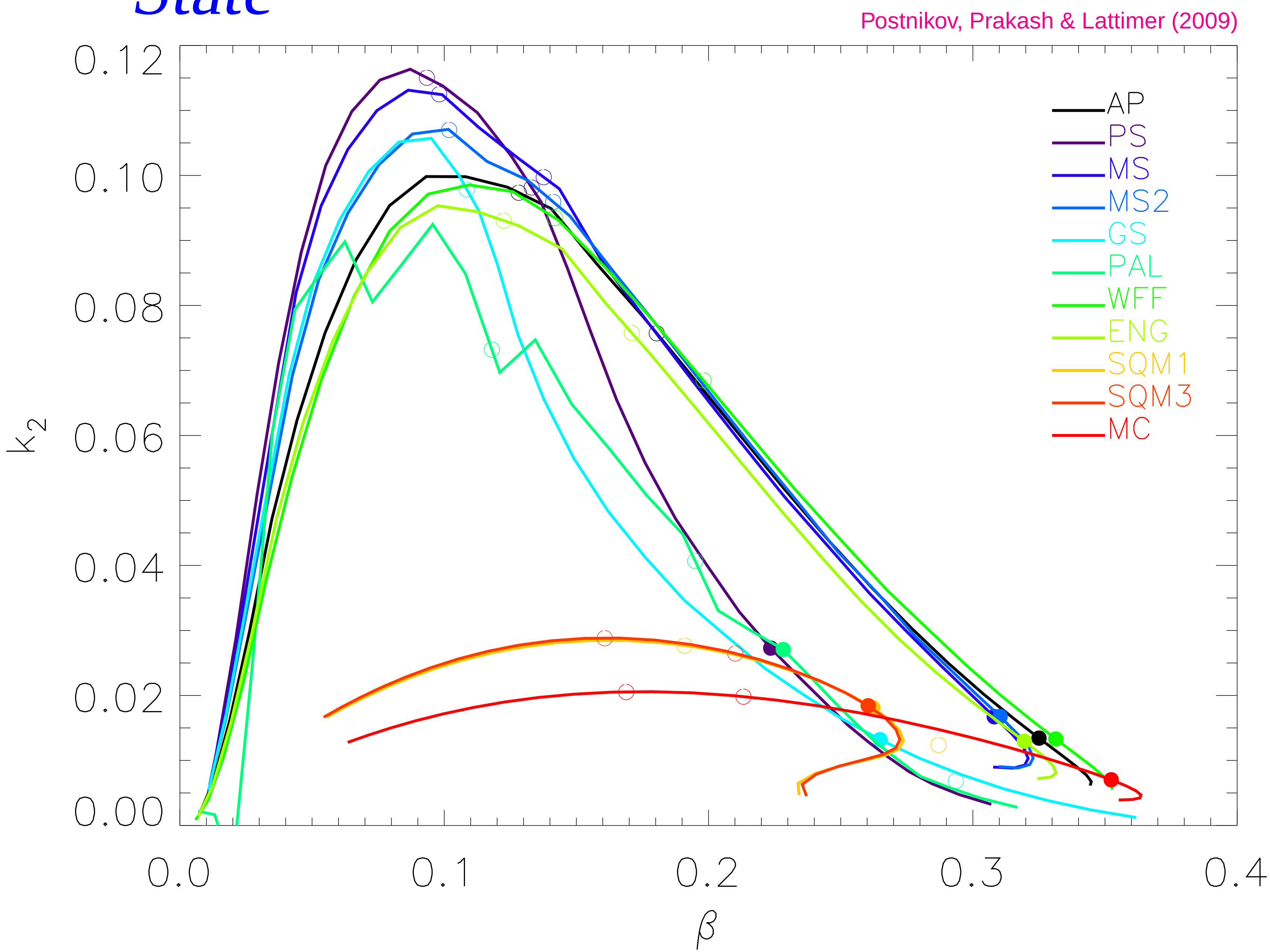


Tidal Love Numbers of Analytic Solutions

Postnikov, Prakash & Lattimer (2009)



Tidal Love Numbers of Realistic Equations of State



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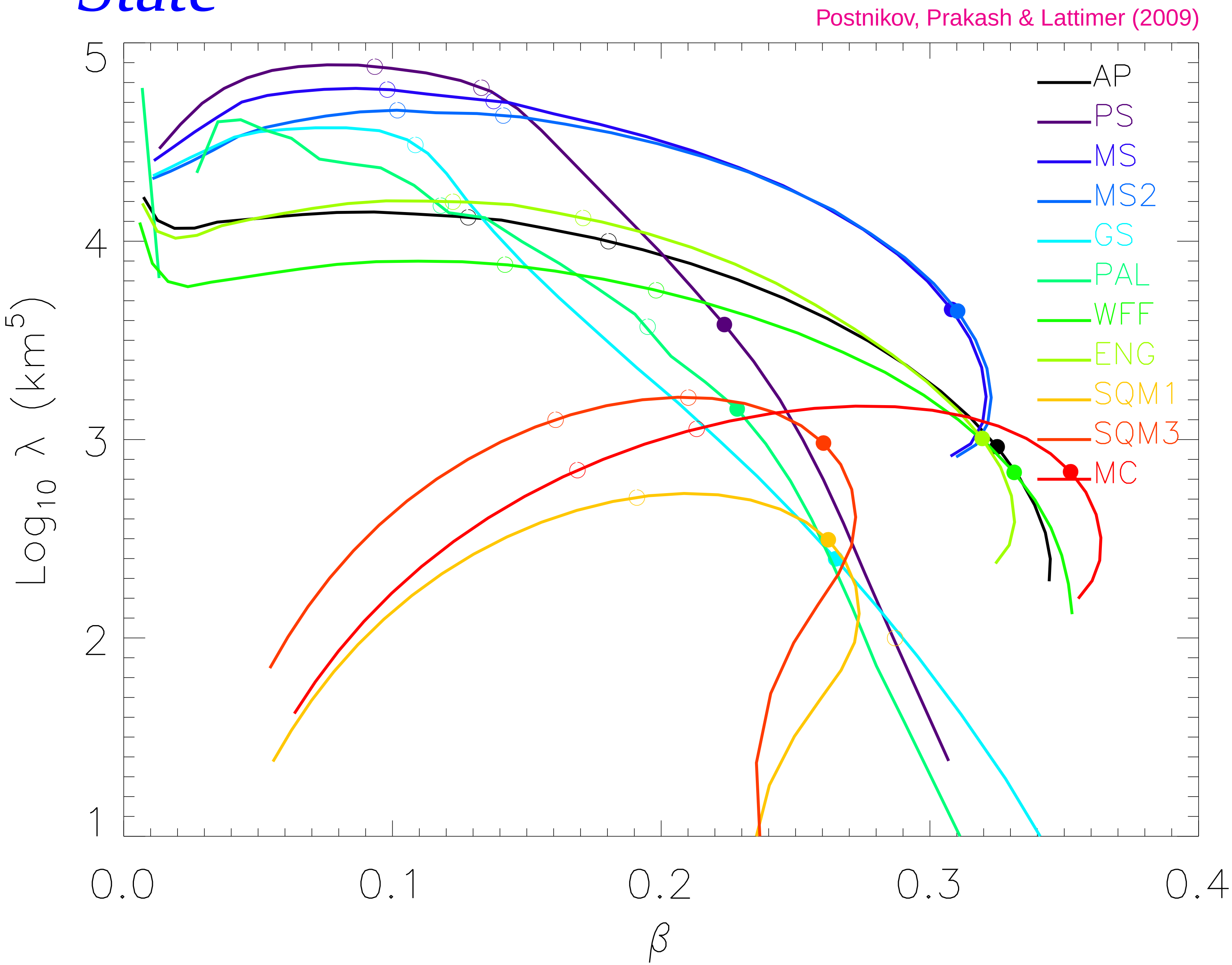
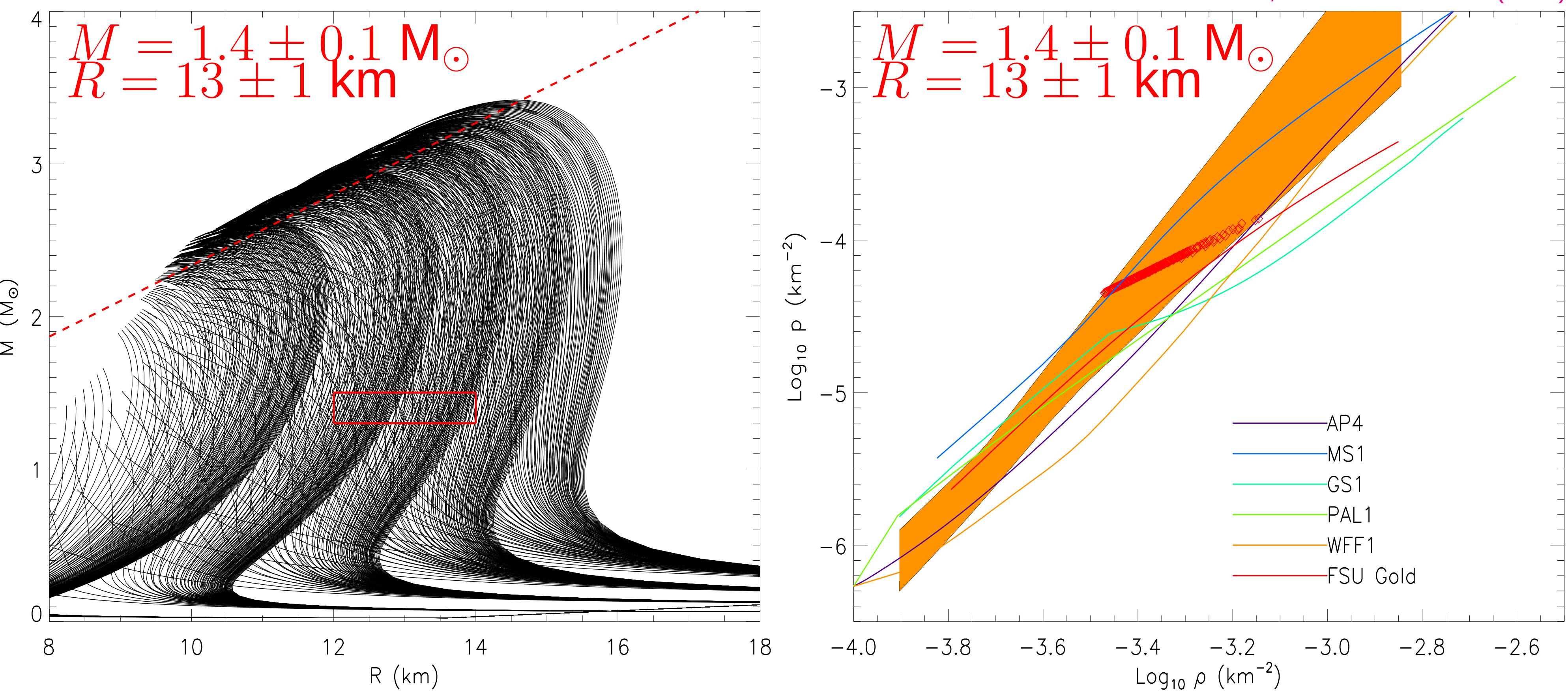


TOV Inversion

How would a simultaneous $M - R$ determination constrain the EOS? Each M-R curve specifies a unique $p - \rho$ relation.

- Generate physically reasonable $M - R$ curves and the $p - \rho$ relations that they specify.
- Generate arbitrary $p - \rho$ relations and compute $M - R$ curves from them; select those $M - R$ curves passing within the error box.

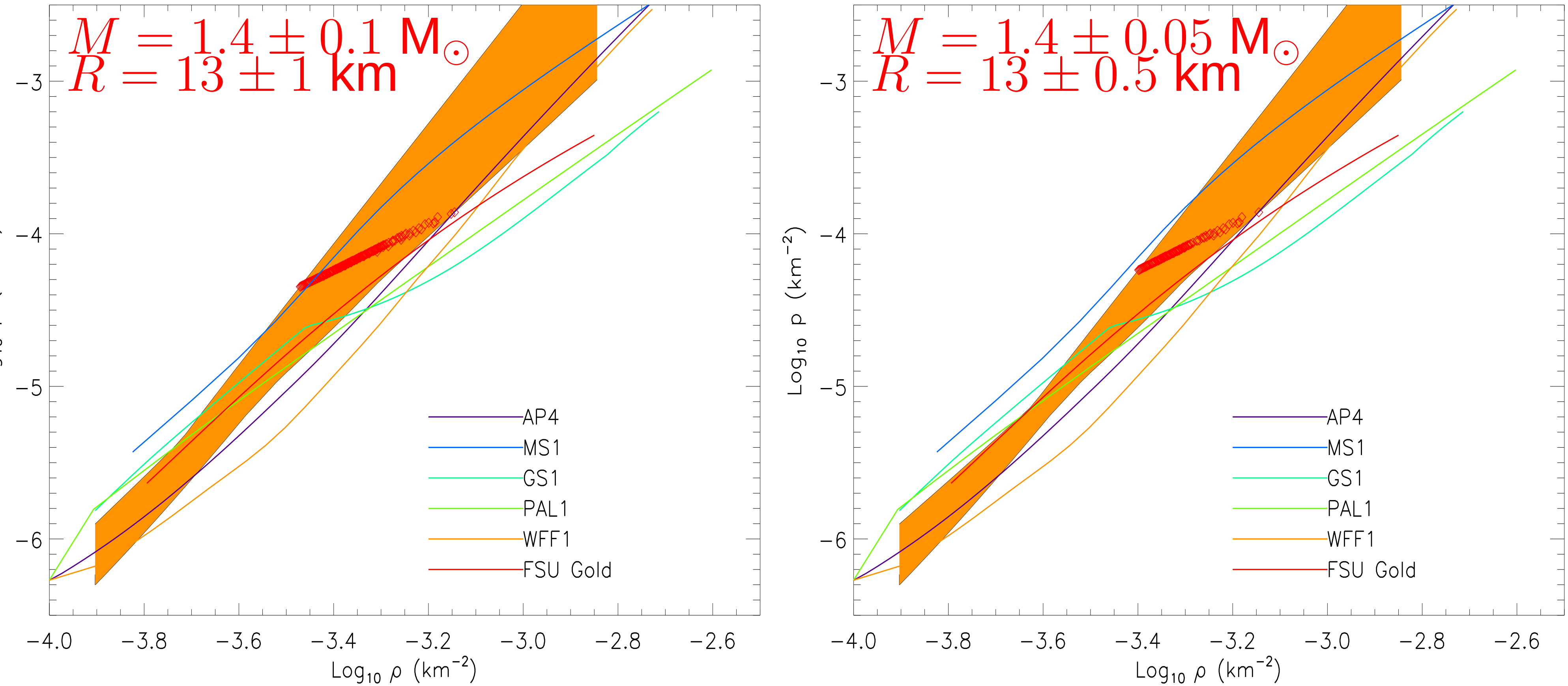
Postnikov, Prakash & Lattimer (2009)



TOV Inversion (cont.)

Dependence on measurement errors

Postnikov, Prakash & Lattimer (2009)



The current uncertainty in the subnuclear EOS introduces significant width to the inferred high-density pressure-density relation.

Conclusions

- **Neutron stars are a powerful laboratory to constrain dense matter physics, especially the symmetry energy and composition at supranuclear densities.**
- **Many aspects of neutron star structure depend on specific equation of state parameters or their density dependence in a model-independent fashion.**
- **Increasing evidence supports the existence of massive neutron stars ($M \gtrsim 1.7$ $M_\odot$), constraining exotic matter.**
- **Many kinds of observations are now available to constrain neutron star radii, although no reliable measures yet exist.**
- **An accurate, simultaneous mass and radius measurement from even one neutron star would provide a significant constraint.**