Fedora GNU/Linux; $\LaTeX\ 2\epsilon$; xfig

Quark matter in neutron stars

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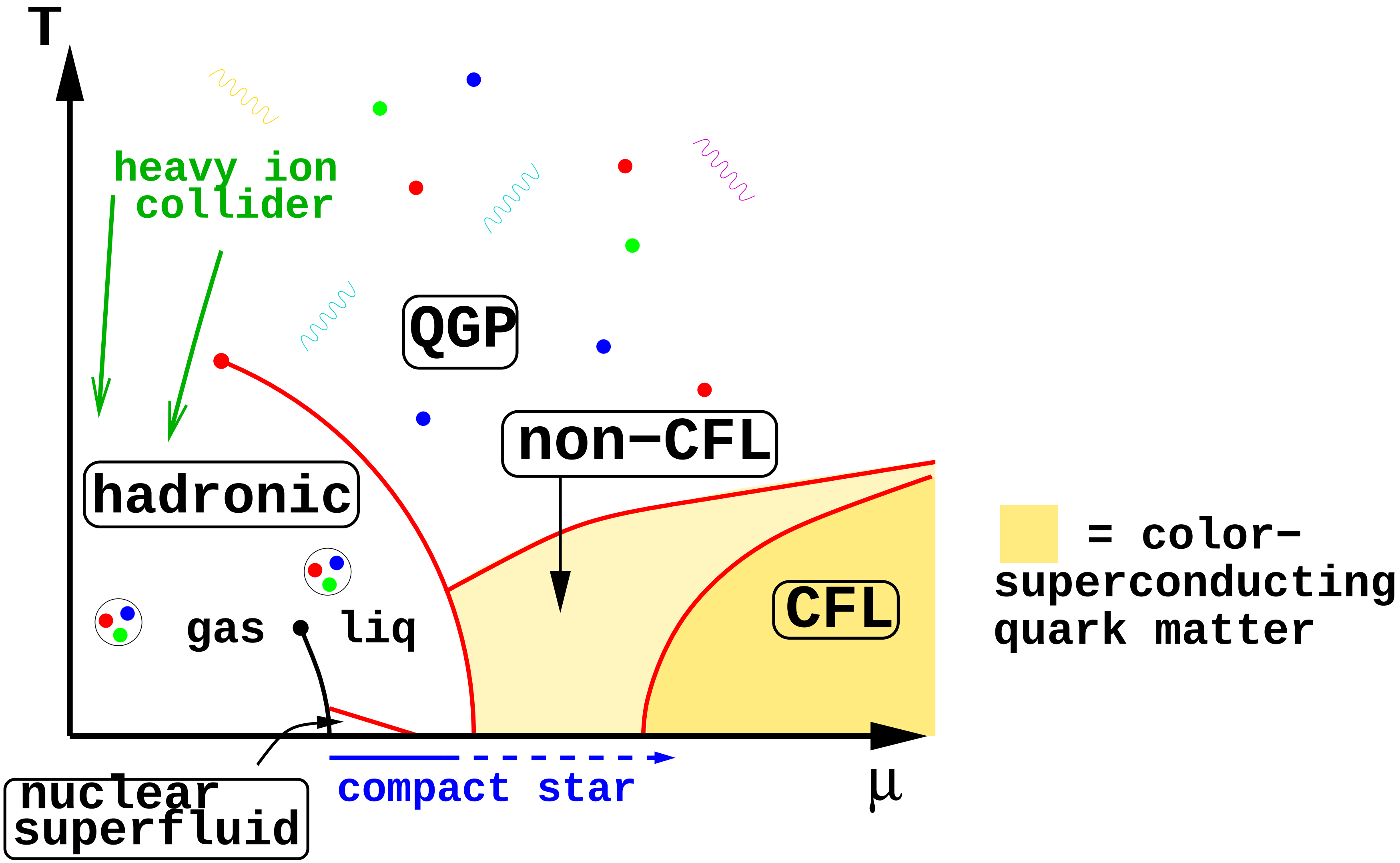
M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, `arXiv:0709.4635` (RMP review)

Conjectured QCD phase diagram



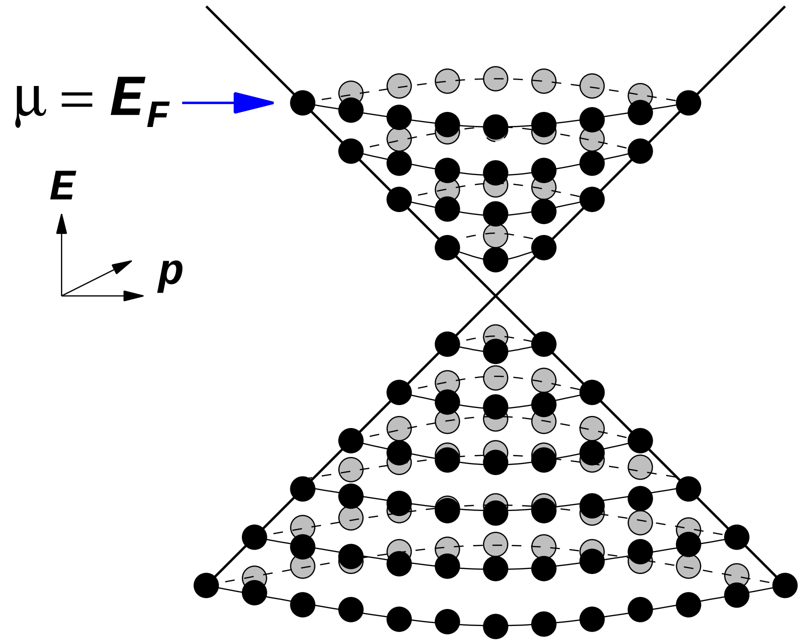
heavy ion collisions: chiral critical point and first-order line

compact stars: color superconducting quark matter core

Cooper pairing in quark matter: color superconductivity

At sufficiently high density and low temperature, there is a Fermi sea of almost free quarks.

Any attractive quark-quark interaction causes pairing instability of the Fermi surface: BCS mechanism of superconductivity.



$$F = E - \mu N \qquad \frac{dF}{dN} = 0$$

QCD quark-quark interaction is attractive in color-antisymmetric channel:

- single gluon exchange
- instanton interaction
- strong coupling: count flux tubes
- confinement is attraction

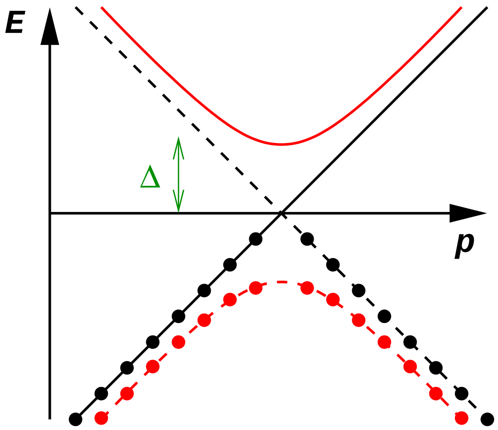
BCS in quark matter: Ivanenko and Kurdgelaidze, Lett. Nuovo Cim. IIS1 13 (1969).

Physical consequences of Cooper pairing

Changes low energy excitations, affecting *transport properties*.

- spontaneous breaking of global symmetries: Goldstone bosons, massless degrees of freedom that dominate low energy behavior. E.g. light pions, superfluidity.
- spontaneous breaking of local (gauged) symmetries: massive gauge bosons, exclusion of magnetic fields (Meissner effect).
- create a gap in fermion spectrum.

Adding a fermion of momentum $\vec{p}$ near the Fermi surface disrupts the condensate in that mode, i.e. breaks the Cooper pair with momenta $\boldsymbol{p}$ and $-\boldsymbol{p}$, costing energy Δ.



Color superconducting phases

Quark Cooper pair: $\langle q^{\alpha}_{ia} q^{\beta}_{jb} \rangle$

color $\alpha, \beta = r, g, b$
flavor $i, j = u, d, s$
spin $a, b = \uparrow, \downarrow$

Each possible BCS pairing pattern P is an 18×18 color-flavor-spin matrix

$$\langle q^{\alpha}_{ia} q^{\beta}_{jb} \rangle_{1PI} = \Delta_P \, P^{\alpha\beta}_{ij\,ab}$$

The attractive channel is:

color antisymmetric [most attractive]
spin antisymmetric [isotropic]
⇒ flavor antisymmetric

So we expect pairing between *different flavors*.

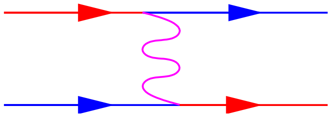
Calculating properties of high-density quark matter

Lattice: "Sign problem"—negative probabilities.

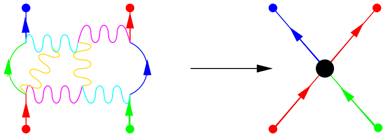
SUSY: Statistics crucial to quark Fermi surface.

large N_c: Large corrections. (Also gravity dual theories.)

pert: Applicable far beyond nuclear density.
Neglects confinement and instantons.



NJL: Model, applicable at low density.
Follows from instanton liquid model.



EFT: Effective field theory for lightest degrees of freedom.
"Parameterization of our ignorance": assume a phase, guess coefficients of interaction terms (or match to pert theory), obtain phenomenology.

Calculations using NJL or weak-coupling QCD

Guess a color-flavor-spin pairing pattern P; to obtain gap Δ_P, calculate free energy Ω (mean-field approx typically), minimize with respect to Δ_P and impose color and electric neutrality

$$\frac{\partial\Omega}{\partial\Delta_P} = 0 \qquad \frac{\partial\Omega}{\partial\mu_i} = 0$$

The pattern with the lowest $\Omega(\Delta_P)$ wins!

1. Weak-coupling methods. First-principles calculations direct from QCD Lagrangian, valid in the asymptotic regime, currently $\mu \gtrsim 10^6$ MeV.
2. Nambu–Jona-Lasinio models, ie quarks with four-fermion coupling based on instanton vertex, single gluon exchange, etc. This is a semi-quantitative guide to physics in the compact star regime $\mu \sim 400$ MeV, not a systematic approximation to QCD.

NJL gives $\Delta \sim 10 - 100$ MeV at $\mu \sim 400$ MeV.

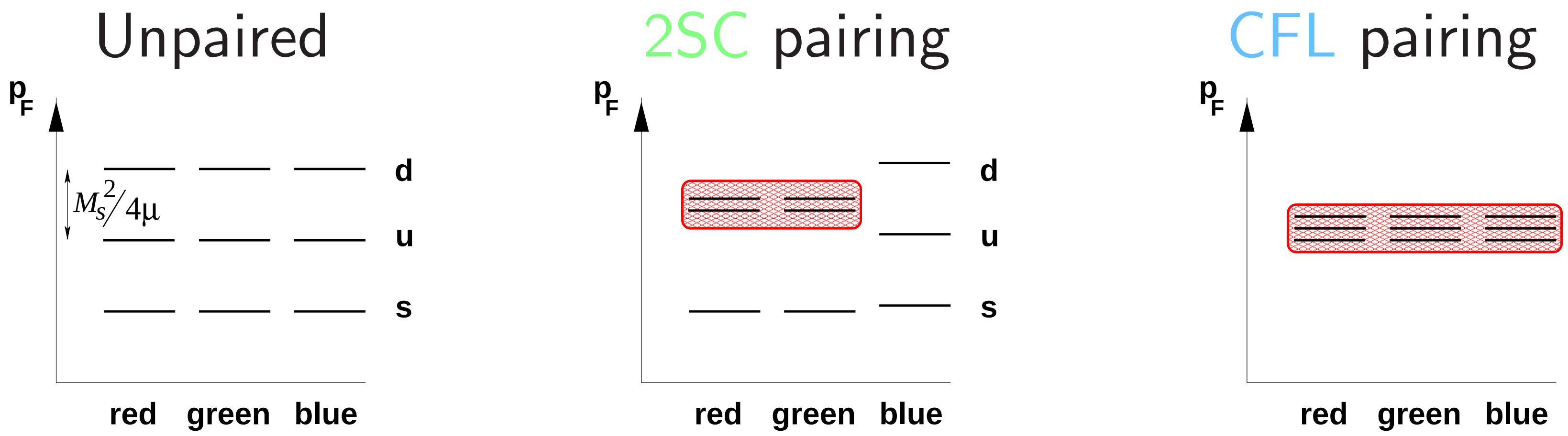
The real world: M_s and neutrality

In the real world (ie neutron star cores) there are three complications.

1. Strange quark mass is not infinite nor zero, but intermediate. It depends on density, and ranges between about 500 MeV in the vacuum and about 100 MeV at high density.
2. Neutrality requirement. Bulk quark matter must be neutral with respect to all gauge charges: color and electromagnetism.
3. Weak interaction equilibration. In a compact star there is time for weak interactions to proceed: neutrinos escape and flavor is not conserved.

So quark matter in a compact star might be CFL, or something else: kaon-condensed CFL, 2SC, 1SC, crystalline,. . .

Cooper pairing vs. the strange quark mass



CFL: Color-flavor-locked phase, favored at the highest densities.

$$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta N} \epsilon_{ijN}$$

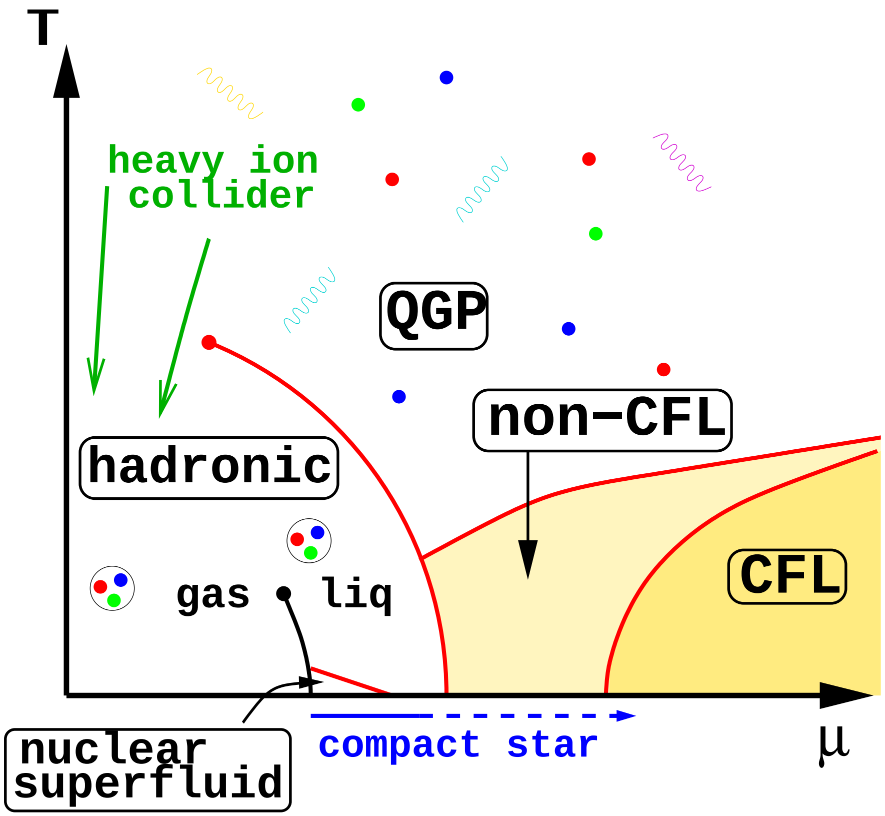
breaks chiral symmetry by a new mechanism: $\langle qq \rangle$ instead of $\langle \bar{q} q \rangle$.

2SC: Two-flavor pairing phase. May occur at intermediate densities.

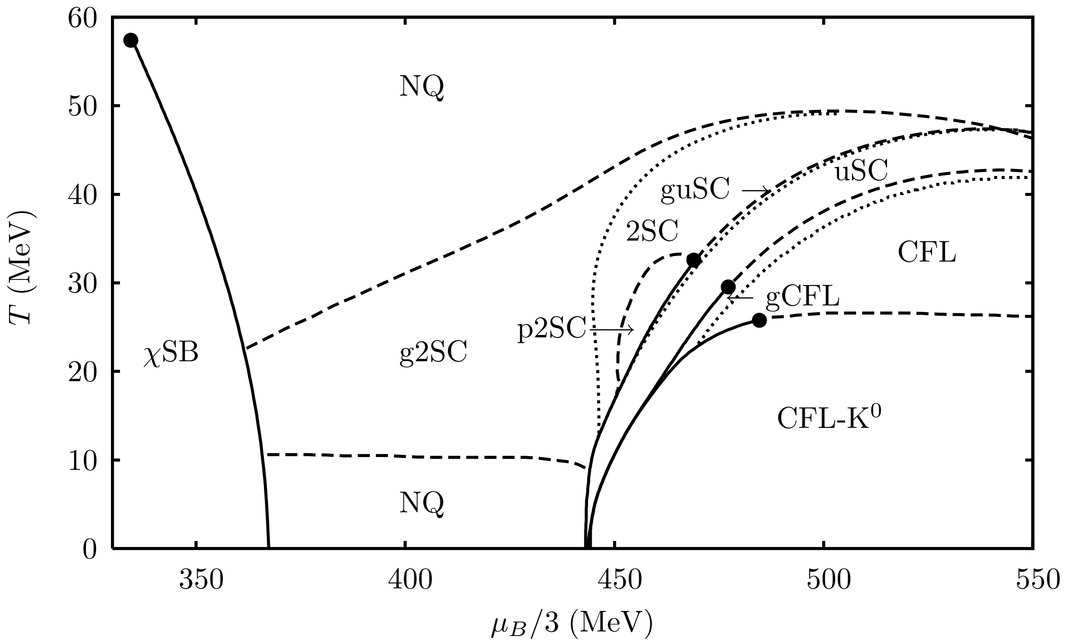
$$\langle q_i^\alpha q_j^\beta \rangle \sim \epsilon^{\alpha\beta 3} \epsilon_{ij3} \sim (rg - gr)(ud - du)$$

or: Exotic non-BCS pairing: LOFF (crystalline phase), p-wave meson condensates, single-flavor pairing (color-spin locking, ~liq ^{3}He-B).

Phases of quark matter, again



NJL model, uniform phases only

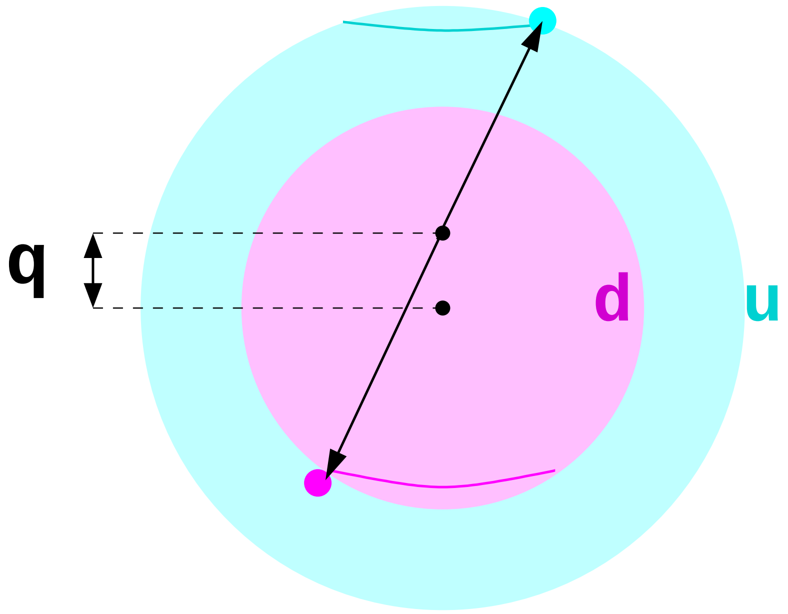


Warringa, `hep-ph/0606063`

But there are also non-uniform phases, such as the crystalline ("LOFF" / "FFLO") phase. (Alford, Bowers, Rajagopal, `hep-ph/0008208`)

Crystalline (LOFF) color superconductivity

When s-wave pairing between different flavors is stressed to near the breaking point, it may be favorable to make pairs with a net momentum, so each flavor can be close to its Fermi surface.



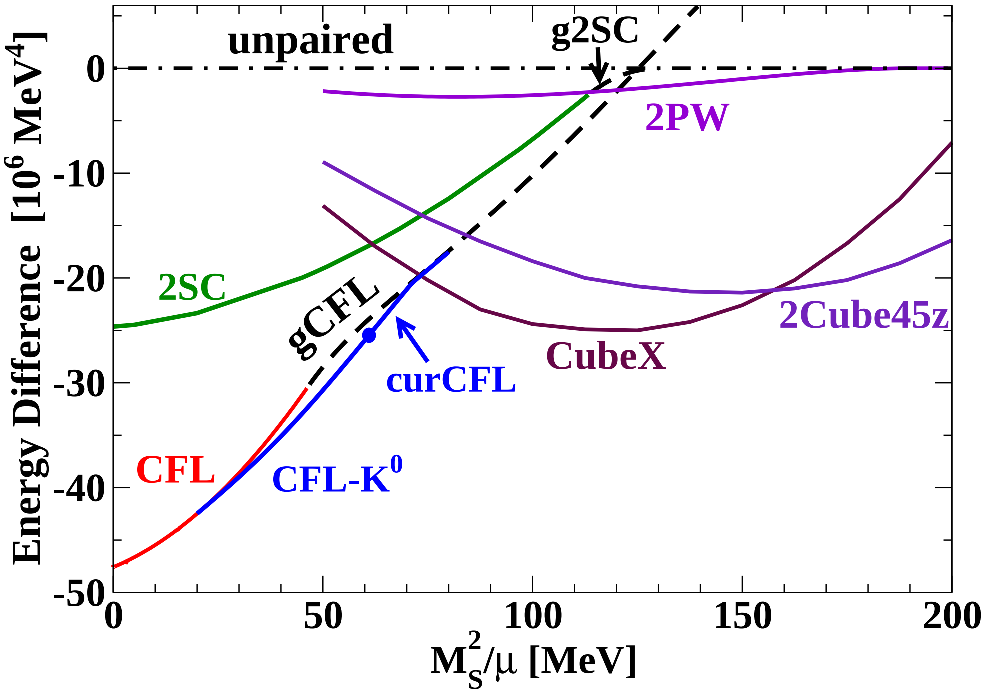
This yields a plane-wave condensate $\Delta(\mathbf{x}) = \Delta_0 \exp(2i\mathbf{q}\cdot\mathbf{x})$.

Two plane waves: $\Delta(\mathbf{x}) = \Delta_0 \cos(2\mathbf{q}\cdot\mathbf{x})$

With three flavors one can combine many plane waves to get crystal structures such as BCC (Rajagopal and Sharma `hep-ph/0605316`).

Free energy comparison of phases

Assuming $\Delta_{\mathrm{CFL}} = 25$ MeV.



CFL-K^0	K^0 condensate
curCFL	K^0 cond current
2PW	LOFF, 2-plane-wave
CubeX	LOFF crystal, G-L approx
2Cube45z	LOFF crystal, G-L approx

(Alford, Rajagopal, Schäfer, Schmitt, `arXiv:0709.4635`)

Curves for CubeX and 2Cube45z use G-L approx far from its area of validity: favored phase at $M_s^2 \sim 4\mu\Delta$ remains uncertain.

Signatures of color superconductivity in compact stars

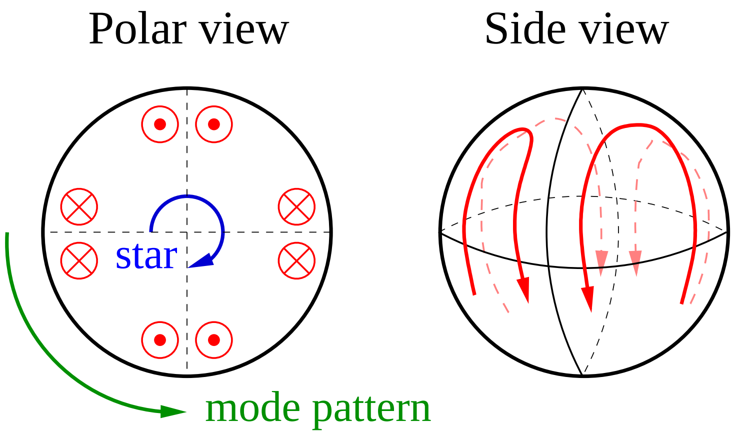
Pairing energy $\begin{cases}\text{affects Equation of state. Hard to detect.}\\ \text{(Alford, Braby, Paris, Reddy, nucl-th/0411016)}\end{cases}$

Gaps in quark spectra and Goldstone bosons $\begin{cases}\text{affect Transport properties:}\\ \text{emissivity, heat capacity, viscosity (shear, bulk),}\\ \text{conductivity (electrical, thermal)\ldots}\end{cases}$

1. Gravitational waves: r-mode instability, shear and bulk viscosity
2. Glitches and crystalline ("LOFF") pairing
3. Cooling by neutrino emission, neutrino pulse at birth

r-modes: gravitational spin-down of compact stars

An r-mode is a quadrupole flow that emits gravitational radiation. It becomes unstable (i.e. arises spontaneously) when a star spins fast enough, and if the shear and bulk viscosity are low enough.



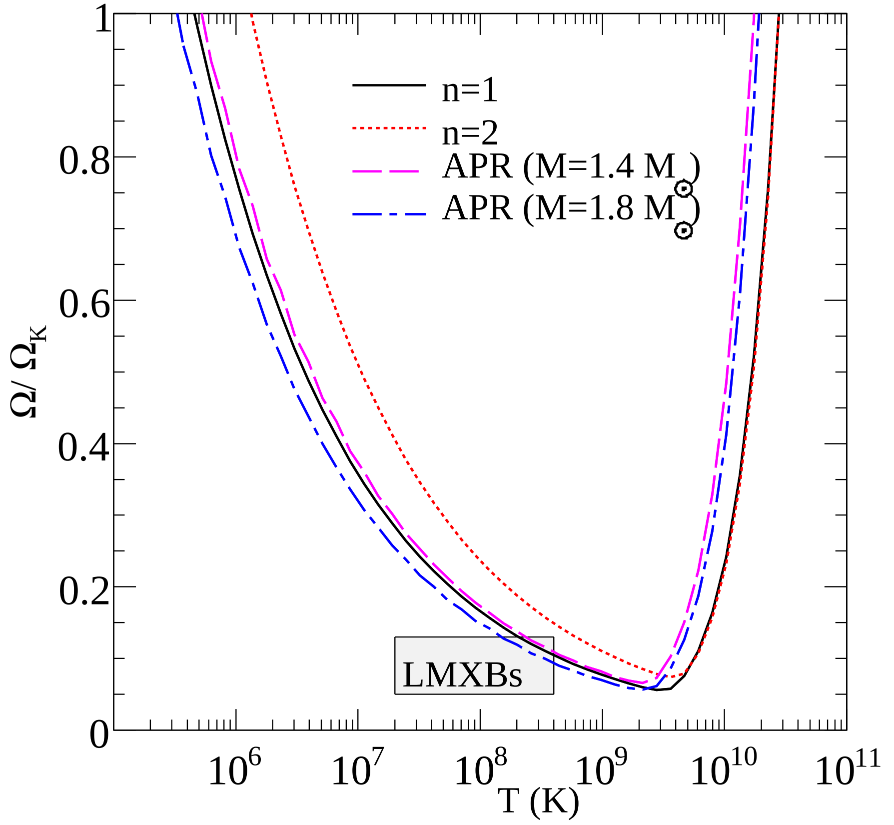
The unstable r-mode can spin the star down very quickly, in days to years (Andersson `gr-qc/9706075`; Friedman and Morsink `gr-qc/9706073`; Lindblom `astro-ph/0101136`).

So if we see a star spinning quickly, we can infer that the interior viscosity must be high enough to damp the r-modes.

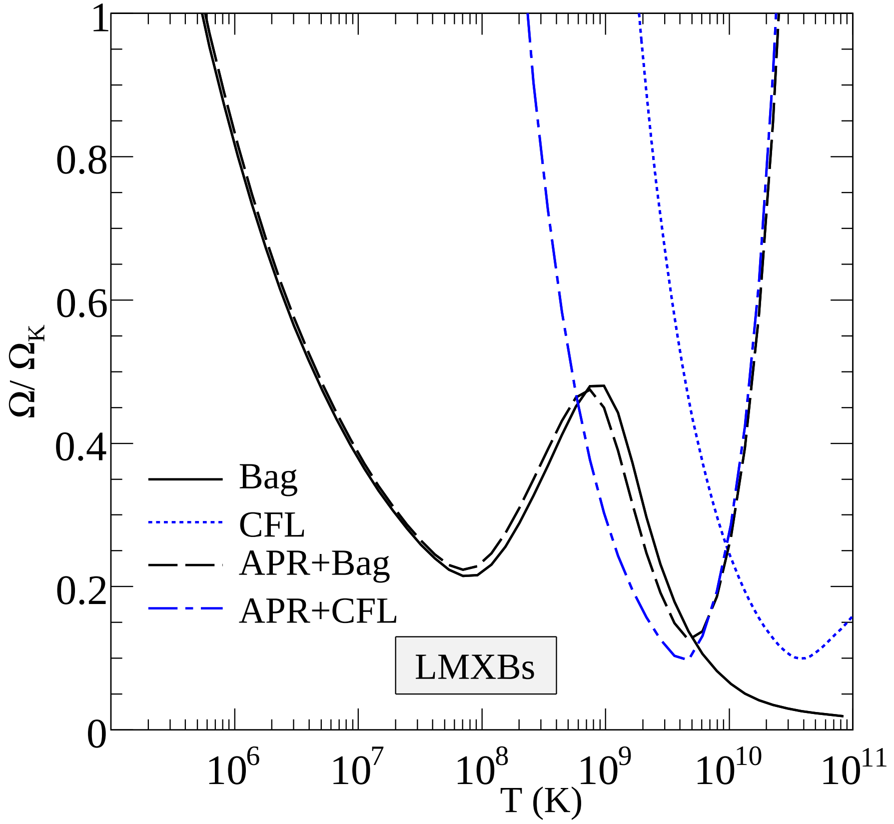
Constraints from r-modes

Regions above curves are forbidden because viscosity is too low to hold back the r-modes.

Nuclear matter

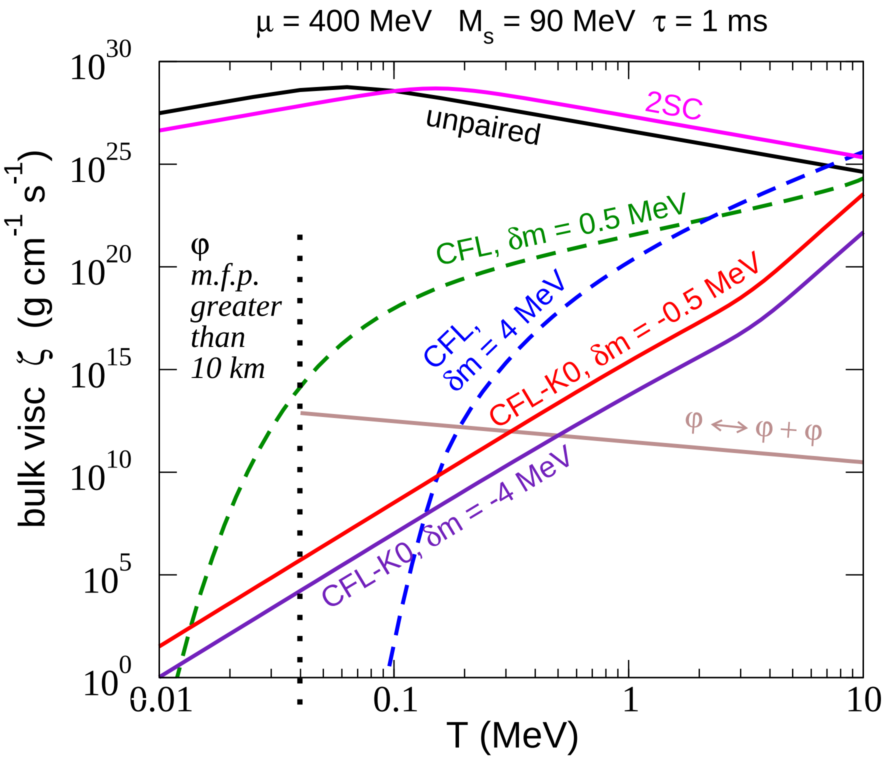


Quark matter



(Jaikumar, Rupak, Steiner `arXiv:0806.1005`)

Bulk viscosity of uniform quark matter phases



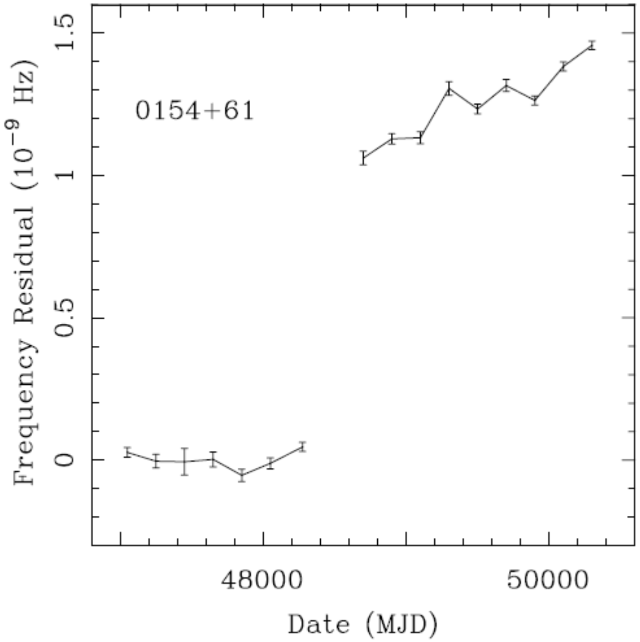
δm is the kaon mass gap
$\delta m > 0$: CFL
$\delta m < 0$: CFL-K^0

Alford, Schmitt, `nucl-th/0608019`; Alford, Braby, Reddy, Schäfer,
`nucl-th/0701067`;
Manuel, Llanes-Estrada,
`arXiv:0705.3909`;
Alford, Braby, Schmitt,
`arXiv:0806.0285`

- Unpaired and 2SC have the largest bulk viscosity, because they have unpaired modes at Fermi surface (large phase space).
- K^0 density $\sim \exp(-\delta\mu/T)$ drops rapidly for $T \lesssim \delta\mu/10$.
- $\delta\mu = m_{K^0} - M_s^2/(2\mu)$ could be anything from negative (kaon condensation) to ~ 10 MeV.
- Superfluid modes ("phonons") alone contribute some bulk viscosity.

Rigidity of quark matter: glitches and grav. waves

Glitch: star's rotation rate suddenly increases. Thought to be due to transfer of ang mom from core to crust as superfluid vortices unpin from some rigid structure in the star.

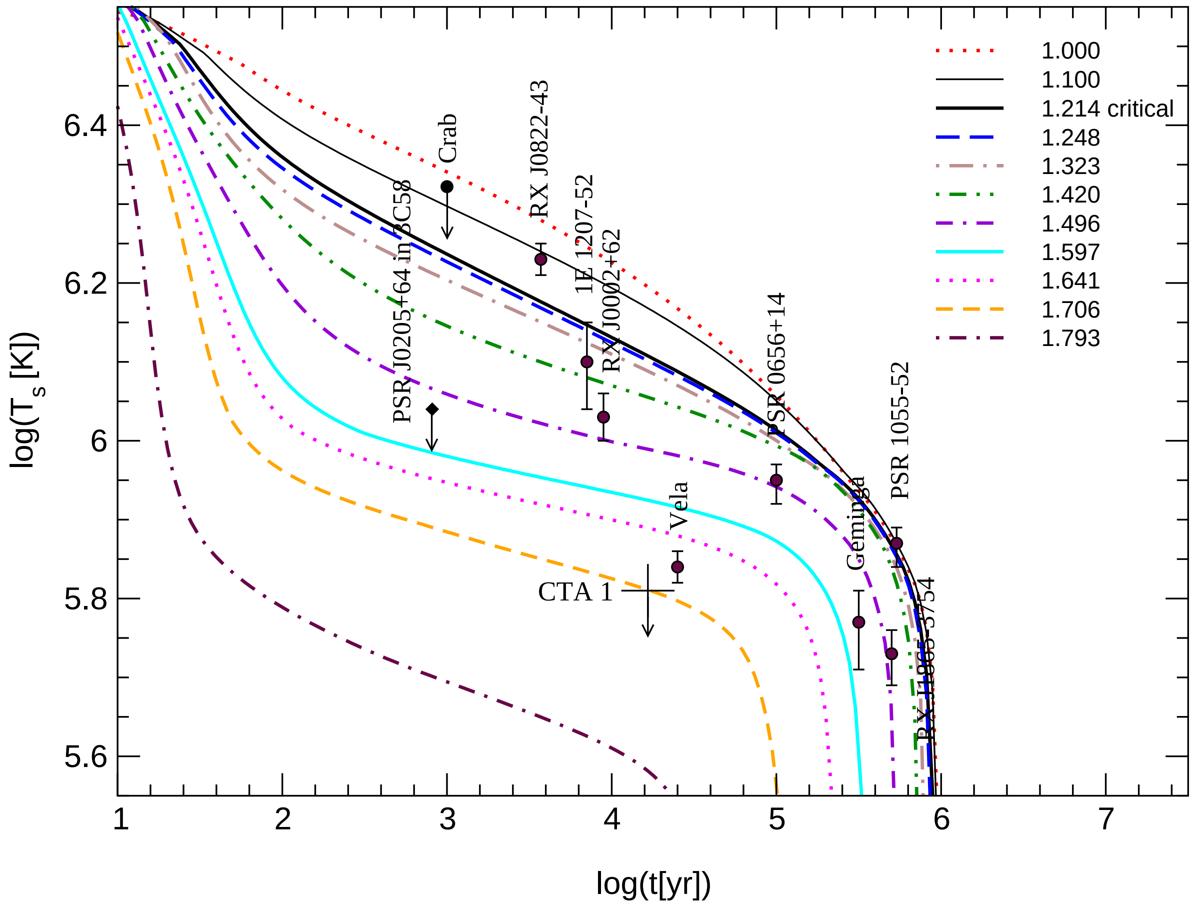


Conventional picture: pinning occurs in "inner crust" where neutron superfluid interpenetrates a lattice of nuclei, with shear modulus $\nu \sim 10^{-4}$-10^{-2} MeV/fm^3.

Alternative scenario: pinning occurs in quark matter core in LOFF phase: superfluid and rigid crystal, shear modulus $\nu \sim 0.5$-20 MeV/fm^3. (Mannarelli, Rajagopal, Sharma `hep-ph/0702021`)

Open questions: pinning force, angular momentum transport time.

Cooling of a neutron star with quark matter core



(Grigorian, Blaschke, Voskresensky, `astro-ph/0411619`)

With 2-flavor color superconductivity, and additional weak pairing of the blue quarks. Can accomodate data with masses ranging from $1.1\ M_\odot$ to $1.7\ M_\odot$.

Looking to the future

- Neutron-star phenomenology of color superconducting quark matter:
    - suprathermal bulk viscosity of quark matter phases (r-mode)
    - detailed analysis of r-mode profiles in hybrid star
    - heat capacity, conductivity and emissivity (neutrino cooling)
    - structure: nuclear-quark interface (gravitational waves?)
    - crystalline phase (glitches) (gravitational waves?)
    - CFL: vortices but no flux tubes
- More general questions:
    - magnetic instability of gapless phases
    - better weak-coupling calculations, include vertex corrections
    - go beyond mean-field, include fluctuations
    - solve the sign problem and do lattice QCD at high density.