A Futuristic Talk: The Barbero-Immirzi "Field" and Inflation.

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A very poor post-doc's picture of K-Inflation
(Mukhanov, Damour)

- Let inflation be driven by purely a Kinetic term.

$$S \propto \int R + K(\phi)X + L(\phi)X^2, \qquad X := \frac{1}{2}(\partial\phi)^2$$

- This leads to a non-trivial density and pressure (in analogy to a perfect fluid with app. 4-velocity)

$$\rho = K \quad + 3\, L \quad ^2, \qquad\qquad = K \quad + \; L \quad ^2$$

- One can rewrite the Friedman Eqs. as D[rho] and D[phi] and then study under what K and L -> inflation sols.

- Specific details of K and L determine theoretical viability

A Shaky Conjecture: K-Inflation in LGG/LQC?

- "Unlike stringy life, loopy life has no natural fields."
- It feels logical, then, to try to explain inflation in LQG in a manner that depends only on scalar fields? (...not!)
- However, we shall see that there is a "natural" scalar field that could arise -> what dynamics? Inflation?
- But just becase phi arises does not mean we have inflation...see eg. the string cosmology debacle:

"...It's amazing that there are so many moduli, yet it is so hard to combine string theory and inflation in a natural way"

(famous string theorist)

LQG could pull ahead!

The Non-Constancy of Physical "Constants"

- Physical constants determine the strength of interactions, playing a critical role in physical theories.
- In LQG, one parameter controls the eigenvalue of the area operator and Hawking radiation...but what if we...

Consider the possibility that the Barbero-Immirzi (BI) parameter is not a constant!!

- Theories that lead to varying physical constants are old. Eg, Brans-Dicke theory. Couple a field to the Einstein-Hilbert action, then G -> G(t,x)

Part 1: Modified Holst Gravity

Promote the BI parameter to a field inside of the action integral.

$$S \propto \int \left[\epsilon_{IJKL} e^I \wedge e^J \wedge F^{KL} + \frac{1}{2}\bar{\gamma} e^I \wedge e^J \wedge F_{IJ}\right]$$

Levi-Civita Tensor

Wedge product

Tetrad

Curvature Tensor

BI scalar $\bar{\gamma} = 1/\gamma$

[One could also add a dynamical (potential) term for the BI scalar, but we won't do this here. You can also couple fermions, etc. -> Simone+Taveras].

The Torsion Constraint

Vary the action with respect to the (a) tetrad e^I; (b) the spin connection w^(KL); (c) the BI scalar

$$(b) \quad \epsilon_{IJKL}\, T^I \wedge e^J + 2\,\bar{\gamma}\, T_{[K} \wedge e_{L]} = e_L \wedge e_K \wedge D\bar{\gamma}$$

Torsion Tensor

The RHS forces the torsion to be non-vanishing

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Amazing parity violation?

$$\mathrm{T}^I = \frac{1}{2}\frac{1}{\bar\gamma^2+1}\left[\epsilon^I{}_{JKL}\partial^L\bar\gamma + \bar\gamma\;\delta^I_{[J}\partial_{K]}\bar\gamma\right] e^J \wedge e^K$$

Effective Field Equations

Inserting the torsion solution into the EOM for the metric (a) and for the BI scalar (c):

$$\mathrm{G}_{\mu\nu} = \kappa T_{\mu\nu} \rightarrow T_{\mu\nu} = \frac{1}{2\kappa}\left[(\partial_\mu\phi)(\partial_\nu\phi) - \frac{1}{2}g_{\mu\nu}(\partial^\sigma\phi)(\partial_\sigma\phi)\right]$$

$$\Box\phi = 0$$

Modified Holst Gravity := GR + Holst Fluid!

upon field redefinition

$$\gamma = \frac{1}{\bar{\gamma}} = \frac{1}{\sinh\left(\phi/\sqrt{3}\right)}$$

No parity violation!

Part 2: Cosmological Solutions

- All GR solutions are preserved for constant BI scalar.
- Simplest cosmological study (flat FRW, scale factor a(t), Hubble Par H, no back-reaction, homogenous-isotropic BI)

$$-3\frac{\ddot{a}}{a} = \frac{3}{2}\frac{\dot{\bar{\gamma}}^2}{\bar{\gamma}^2+1}$$

⟵ correction to Friedman Eqs.

$$\ddot{\bar{\gamma}} + 3H\dot{\bar{\gamma}} = \frac{\bar{\gamma}}{\bar{\gamma}^2+1}\dot{\gamma}^2$$

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$$a(t) \propto L_0^{2/3}(t-t_0)^{1/3}$$ ← Same as pressureless, perfect fluid with stiff EOS (w= 1)

$$\bar{\gamma} \propto (t-t_0)^{2/3} + (t-t_0)^{-2/3}$$ ← BI parameter tends to zero as t -> to and t -> infinity.

Inflating Holst Gravity

1. Spatially-varying BI scalar?

Compare to comoving, perfect fluid with EOS $p = \omega\rho$

$$\omega(t) = \frac{a^2\dot{\bar{\gamma}}^2 - (\partial_i\bar{\gamma})(\partial^i\bar{\gamma})}{a^2\dot{\bar{\gamma}}^2 + (\partial_i\bar{\gamma})(\partial^i\bar{\gamma})} \sim 1 - 2\frac{(\partial_i\bar{\gamma})(\partial^i\bar{\gamma})}{a^2\dot{\bar{\gamma}}^2}$$

$\dot{\bar{\gamma}}/\partial\bar{\gamma} \ll 1$

In the limit of time-indep BI scalar, w = -1 !!! -> Inflation?

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2. UV Completion?

$$S_{eff} \propto \int d^4x\sqrt{-g}\left[R + \frac{3}{2}\frac{\dot{\bar{\gamma}}^2}{\bar{\gamma}^2+1} - \frac{9}{4}\frac{\dot{\bar{\gamma}}^4}{\left(\bar{\gamma}^2+1\right)^2} + \dots\right]$$

from R squared

K-inflation! non-trivial K drives inflation w/out V

Conclusions and Future Work

- Modified Holst Gravity (ie, a varying BI scalar in the Holst action) is equivalent to GR in the presence of a massless scalar fluid related to the BI scalar.
- Natural relaxation mechanism for the BI parameter to flow toward the BH thermodynamics value?
- Inflationary solutions for an appropriate BI scalar or upon UV completion? Many details remain unknown.
- New clock to measure e-folding with during super-inflation?

Solving the Torsion Constraint

Solve the torsion equation (b) using the Rovelli-Perez operator $\mathrm{p}_{IJ}{}^{KL} := \delta_I^{[K}\delta_J^{L]} - \frac{1}{2}\bar{\gamma}\,\epsilon_{IJ}{}^{KL}$

$$(b) \quad p^{IJ}{}_{KL}\, D\,(e_I e_J) = \frac{1}{2} e_I e_J\, \epsilon^{IJ}{}_{KL}\, D\bar{\gamma}$$

$$\mathrm{T}^I = \frac{1}{2}\frac{1}{\bar{\gamma}^2+1}\left[\epsilon^I{}_{JKL}\partial^L\bar{\gamma} + \bar{\gamma}\,\delta^I_{[J}\partial_{K]}\bar{\gamma}\right] e^J \wedge e^K$$

Torsion disappears if the BI scalar is a constant

-> we recover GR!

Evolution of the BI scalar

Solve equation (c) for $\bar{\gamma}$ assuming the FRW background.

Int. constant

$$\frac{\dot{\bar{\gamma}}^2}{\bar{\gamma}^2 + 1} = \frac{L_0^4}{a^6} \rightarrow \bar{\gamma} = \sinh\left[\int \frac{L_0^2}{a^3(t)} dt\right]$$

- Near the singularity, a -> 0, so $\bar{\gamma}$-> infinity, which means the BI parameter -> 0!
- Could this be a sign of some sort of "asymptotic freedom"?

Evolution of the Scale Factor

Solve equation (a) for a(t), assuming the found BI scalar.

$$a(t) \propto L_0^{2/3}(t - t_0)^{1/3}$$

Int. constant

Same as pressureless, perfect fluid with stiff EOS (w= 1)

$$\bar{\gamma} \propto (t - t_0)^{2/3} + (t - t_0)^{-2/3}$$

BI parameter tends to zero as t -> to and t -> infinity.

Can we inflate LQG?

But there are a few problems with this picture:

- P: The BI scalar does not tend to the BH thermo. value.
  R: Add a potential? More general BI scalar?
- P: The BI scalar tends to zero near the singularity -> Continuous area spectra in quantum regime?
  R: Study quantization of Mod. Holst more carefully.
- P: A stiff EOS will never lead to inflation.
  R: More general BI scalar? Higher-order curvature terms?

UV Completion?

Reinsert solution to the action to obtain an effective S:

$$S_{eff} \propto \int d^4x \sqrt{-g} \left[R - \frac{3}{2} \frac{1}{\bar{\gamma}^2 + 1} \left(\partial_\mu \bar{\gamma} \right) \left(\partial^\mu \bar{\gamma} \right) \right]$$

K-inflation is a model that predicts inflation without a potential, but with a non-trivial Kinetic term that contains both quadratic and quartic derivatives of the field!

Modified Holst gravity can be mapped to an inflationary solution in the K-inflation scenario upon UV completion, even for space-independent BI scalars.

Example

Let us add an Ricci scalar squared term to the Holst S:

$$S_{eff} \propto \int d^4x\sqrt{-g}\left[R + \frac{3}{2}\frac{\dot{\bar{\gamma}}^2}{\bar{\gamma}^2+1} - \frac{9}{4}\frac{\dot{\bar{\gamma}}^4}{(\bar{\gamma}^2+1)^2} + \ldots\right]$$

- The difference in sign between the quadratic and quartic terms would suggest an inflationary solution in the K-inflation model.
- But how do we know what terms to add to S?
- Also, an R-squared term also produces other interaction terms that could spoil the inflationary solution.

Road Map

- Part 1: Modified Holst Gravity: action, variation, torsion, modified field equations and equations of motion.
- Part 2: K-Inflationary LQC: A shaky conjecture...

Disclaimer 1: I shall assume an LQG/C audience, thus a knowledge of GR, the Holst action and exterior calculus shall also be assumed ->
If I baffle you, please interrupt me!

Disclaimer 2: This will be a mostly **mathematical** talk ->
limited use of "twiddle math."