

Loop Quantum Cosmology: Phenomenological Implications (Physics of Singularity Resolution)

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Workshop on LQC, IGC, Penn State

Based on several works of various groups including *Bruni, Cailleteau, Corichi, Maartens, Parisi, Sami, PS, Tsujikawa, Vandersloot, Vereshchagin, Vidotto, Wands ...*

Outline

- Motivation and Caveats
- A bird's eye view of some features of New physics
 - Einstein static model
 - Bounce with magnetic fields
 - Cyclic models
- (Not so) New cosmological singularities in GR
- General analysis of singularities and some insights
- Conclusions and lessons so far

Introduction

- Careful studies of physical phenomena are invaluable tools to understand the strengths and weaknesses of the underlying theory. This becomes all more important for quantizations such as LQC (or those inspired by it) as we do not yet have a link with the full theory.
- One of the expectations is that physical viability will provide constraints on the theory and various ambiguities. This expectation comes out to be true (many times). **Examples:** Phenomenology was first to signal severe problems with old quantization in LQC. Demanding correct IR and UV limits selects a single choice (Improved dynamics or sLQC) from a large class of loopy quantizations. **All which is mathematically possible may be not be viable or relevant in physics.**
- We have very few examples in which a consistent quantization has been performed, physics completely understood using effective equations and numerical simulations used to confirm all the results ($k = 0$ model with massless scalar with $\pm\Lambda$ and pre-inflationary phase, $k = 1$ model with massless scalar). **All these are important to trust any model.** This leads to many caveats in other class of works which should be considered preliminary.
- **Caveats:** Focus on quantum geometric modifications to the gravitational part only. No inclusion of higher order quantum effects resulting from state dependent properties. States assumed to describe large classical universe at late times.

Hamiltonian Constraint (flat model)

$$C_{\text{grav}} = - \int_{\mathcal{V}} d^3x N \varepsilon_{ijk} F_{ab}^i (E^{aj} E^{bk} / \sqrt{|\det E|})$$

Procedure: Express C_{grav} in terms of elementary variables and their Poisson brackets

– Classical identity of the phase space:^a

$$\varepsilon_{ijk} (E^{aj} E^{bk} / \sqrt{|\det E|}) \longrightarrow \text{Tr}(h_k^{(\mu)} \{h_k^{(\mu)-1}, V\} \tau_i)$$

(Peak tied to the fiducial volume)

– Express field strength in terms of holonomies and quantize.

Leads to two types of quantum modifications:

(i) Curvature modifications from field strength. Solely responsible for bounce at $\rho = \rho_{\text{crit}} \sim 0.4\rho_{\text{Pl}}$.

(ii) Inverse triad corrections (also for the matter part). **Not tied to any curvature scale in the flat model. Lack of predictive power.**

Some Features of New Physics:

- Quantum dynamics described by an effective Hamiltonian (improved dynamics) (Taveras's talk)

$$\mathcal{H}_{\text{eff}} = -\frac{3}{8\pi G\gamma^2} \frac{\sin^2(\lambda\beta)}{\lambda^2} V + \mathcal{H}_{\text{matt}}$$

Leads to modified Friedman dynamics

$$H^2 = \frac{8\pi G}{3} \rho \left(1 - \frac{\rho}{\rho_{\text{crit}}}\right)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \rho \left(1 - 4\frac{\rho}{\rho_{\text{crit}}}\right) - 4\pi G P \left(1 - 2\frac{\rho}{\rho_{\text{crit}}}\right)$$

Conservation law is unmodified: $\dot{\rho} = 3H(\rho + P)$.

- Consistency and correct UV and IR limit in general selects this quantization uniquely (Corichi, PS (08)). Also for curved models (PS, Vidotto (08)). Rich physical and phenomenological applications. Those not covered in the talk include^{abcde}

^aScaling solutions: PS (06)

^bInflationary models: Zhang, Ling (07); Copeland, Mulryne, Nunes, Shaeri (07,08)

^cTachyon & Quintom Models: Sen (06); Wei, Zhang (07); Xiong, Qiu, Cai, Zhang (07)

^dCovariant effective action and modified gravity scenarios: Olmo, PS (08)

^eFeatures of big bounce: Mielczarek, Szydlowski, ... (08)

Einstein Static Universe

- An initial singularity free closed FRW model with a positive cosmological constant in GR. Leads to Emergent Universe scenario: initial state for the past eternal inflation. Universe originates as an Einstein static and evolves to inflate and lead to a “hot big bang” (free) standard model.
→ It may be possible to construct a singularity free inflationary spacetime.
- **Problem:** Einstein static model is unstable to homogeneous perturbations. Emergent universe scenarios are heavily fine tuned.
- However, we expect that gravity is modified at high curvatures and it will be naive to expect no change in the properties of the phase space and existence of critical points.
- Quantum geometry modifies the scenario.^a Consider the model with a Λ and matter. Analysis of dynamical equations results in two critical points allowing static solutions. **New critical point admits Einstein static even without cosmological constant.** (However, the critical point is unstable).

For $\Lambda > 6.6\pi M_p^2$, and matter obeying strong energy condition $w > -1/3$ there always exist a stable (centre) critical point.

→ Can we have a viable model of the early universe?

Anisotropic Model: Cosmological Magnetic Fields

Existence of bounce in anisotropic models is an important issue. Shown for matter with vanishing anisotropic stress (massless scalar + radiation). Anisotropies bounded and shear conserved. ^a **Effective Hamiltonian:**

$$\mathcal{H}_{\text{eff}} = -\frac{p_1 p_2 p_3}{\kappa \gamma^2 \Delta} (\sin(\bar{\mu}_2 c_2) \sin(\bar{\mu}_3 c_3) + \dots) + p_1 p_2 p_3 \rho$$

For matter with no anisotropic stress: $p_i c_i - p_j c_j = \gamma V_o \alpha_{ij}$, $\alpha_{ij} = \text{constt.}$

Does the bounce survive on including matter with anisotropic stress?

An important question since in the early Universe cosmological magnetic fields can potentially affect the scenario. Consider a model with magnetic field stress tensor^b

$$\mathcal{H}_{\text{mag}} = \frac{1}{2} a^3 (a_1^2 B^1{}^2), \quad (B^2 = B^3 = 0)$$

$$H_1 + H_2 = \gamma_{12}/a^3, \quad H_1 + H_3 = \gamma_{13}/a^3, \quad H_2 - H_3 = (\gamma_{12} - \gamma_{13})/a^3$$

Depending on whether $a_2 = a_3$, we have an axisymmetric ($a_1 \rightarrow 0$) or non axisymmetric approach to the singularity ($a_1 \rightarrow 0$ and a_2 or $a_3 \rightarrow 0$)

In both cases, LQC predicts a bounce with bounded anisotropies. However, shear is not conserved.

The Cyclic Model

- An alternative to inflationary spacetime (Steinhardt, Turok, Khoury (2001)). Important insights on the behavior of the universe and its homogenization (due to ultra-stiff matter) near the singularity.
- **An open problem:** What happens when the branes collide? Can quantum gravity resolve the singularity? (Pretorius's Talk)
Effectively we need to consider the dynamics with the cyclic potential

$$V = V_o(1 - e^{-\sigma_1\phi})exp(-e^{-\sigma_2\phi})$$

Brane collision corresponds to $\phi \rightarrow -\infty$. In the classical theory, the Hubble diverges and **there is no turn around of the radion field.**

$$\dot{\phi}^2 = \frac{3}{4\pi G} H^2 - 2V$$

($V < 0$ in the region of interest. $\dot{\phi}$ can not vanish!)

As the radion rushes to the bottom of the potential, the energy density (and curvature) quickly become super-Planckian (for realistic choices of parameters). The quantum gravity effects may modify the scenario in a non-trivial way.

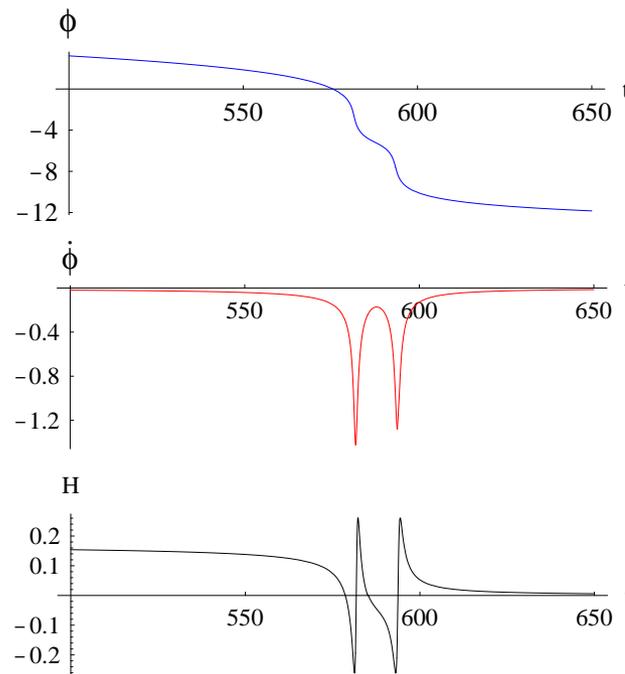
Can this singularity be resolved in LQC?

Isotropic Loop Quantum Cyclic model

Use the modified Friedman dynamics to analyse Cyclic potential.^a Ignore anisotropies for simplification.

As the field rolls down the potential, $\rho \rightarrow \rho_{\text{crit}}$ and there is a bounce. The singularity is resolved **but the field does not turn around**. At the bounce

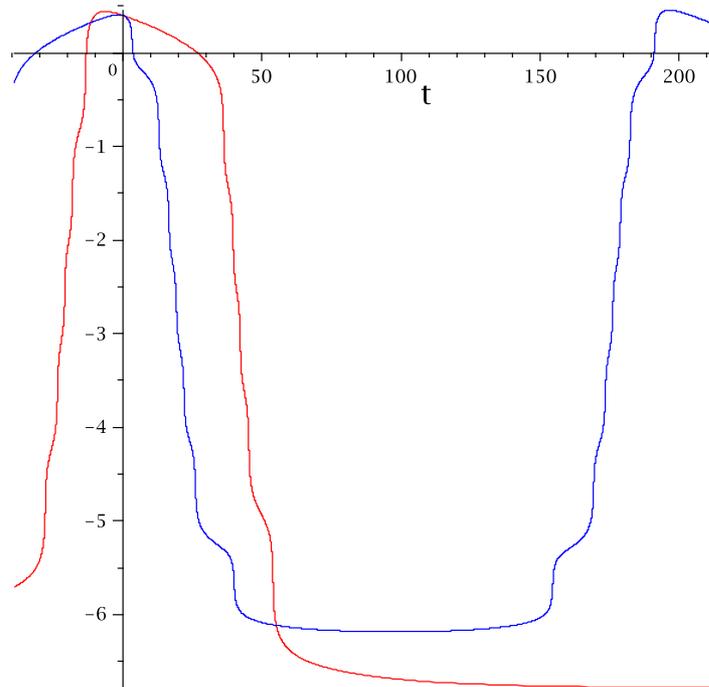
$$\dot{\phi}^2 = \rho_{\text{crit}} - V, \quad V < 0 \Rightarrow \dot{\phi} \neq 0$$



No viable loopy cyclic model (atleast in the isotropic case).

A more complete Loop Quantum Cyclic Model

- Situation changes if we do not simplify and include anisotropies.^a Consider anisotropies using Bianchi-I construction ([Wilson-Ewing's Talk](#))
- Results: The universe bounces at Planck scale, anisotropies remain bounded and the field turns around!



A non-singular viable alternative to inflation possible.

^aCailleteau, PS, Vandersloot (2008)

Is singularity resolution generic?

Any counter examples?

Types of Singularities

- **Big Bang/Big Crunch:** The scale factor vanishes in finite time. Energy density, pressure blow up. All of the curvature invariants diverge. Inevitable fate of the matter satisfying null energy condition (except Λ): $\rho + P \geq 0$.

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- **Type III singularities:** The universe rips suddenly. Singularity occurs at a finite value of scale factor. Curvature invariants diverge.
- **Type IV Singularity:** Occurs at a finite value of scale factor. Energy density and pressure remain finite. Curvature invariants are bounded. However, curvature derivatives diverge.

Various works on future singularities

- [Sami, PS & Tsujikawa \(2006\)](#): Showed that the Type I big rip singularity occurring in certain phantom models ($w < -1$) can be successfully resolved in LQC.

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- PS (2008): Generalized model including all types of singularities. Shows above results to be a special case in a large class.

Modeling singularities

Let us consider a dark energy model based on the ansatz: ^a

$$P = -\rho - f(\rho), \quad f(\rho) = \frac{AB\rho^{2\alpha-1}}{A\rho^{\alpha-1} + B}$$

Using the conservation law (which is unmodified also in LQC):

$$\rho = \left(-\frac{A}{B} \pm \left(\frac{A^2}{B^2} - 6(\alpha - 1)A \ln \left(\frac{a}{a_o} \right) \right) \right)^{1/(1-\alpha)}$$

We probe the dynamics using Friedman and Raichaudhuri equations in classical as well quantum theory.

Modified Raichaudhuri Equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \rho \left(1 - 4\frac{\rho}{\rho_{\text{crit}}} \right) - 4\pi G P \left(1 - 2\frac{\rho}{\rho_{\text{crit}}} \right)$$

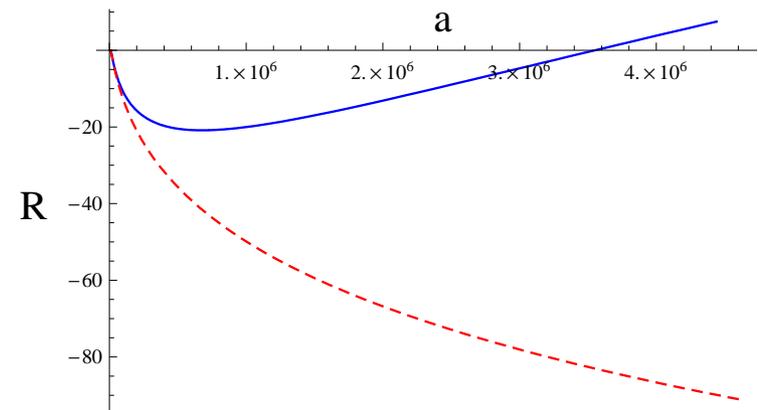
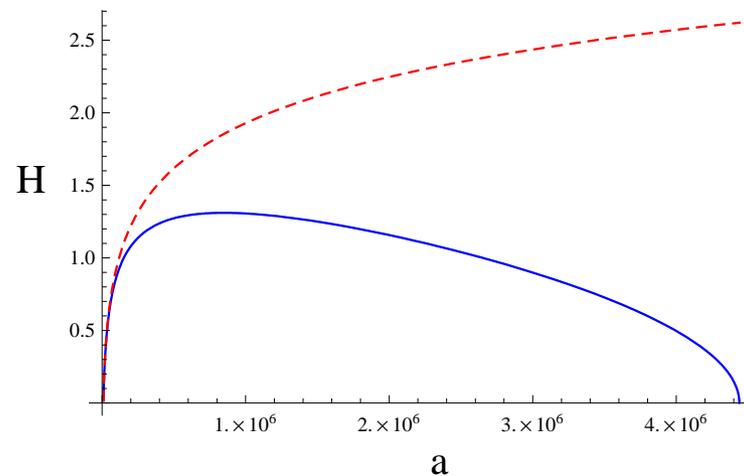
At $\rho = \rho_{\text{crit}}$, $\ddot{a}/a = 4\pi G(1 + w)\rho_{\text{crit}}$

For $w > -1$, universe bounces and for $w < -1$ it recollapses.

Type I Singularity and its Resolution

Parameters: $3/4 < \alpha < 1$, $A > 0$.

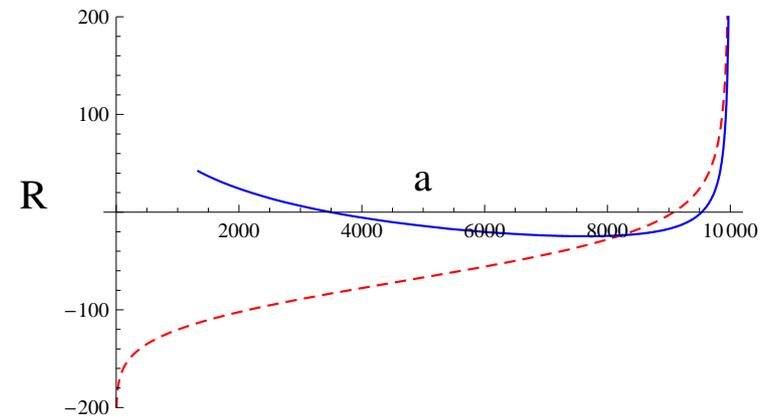
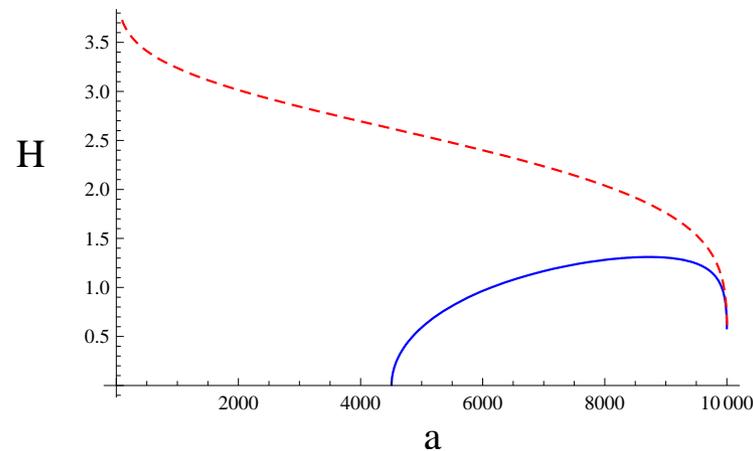
Classically, the ρ , Hubble diverge as $a \rightarrow \infty$ in finite time. Curvature invariants diverge.



Classical big rip is generically avoided.

Type II Singularity and its Non-Resolution

A necessary condition to have a sudden singularity is $A/B < 0$.



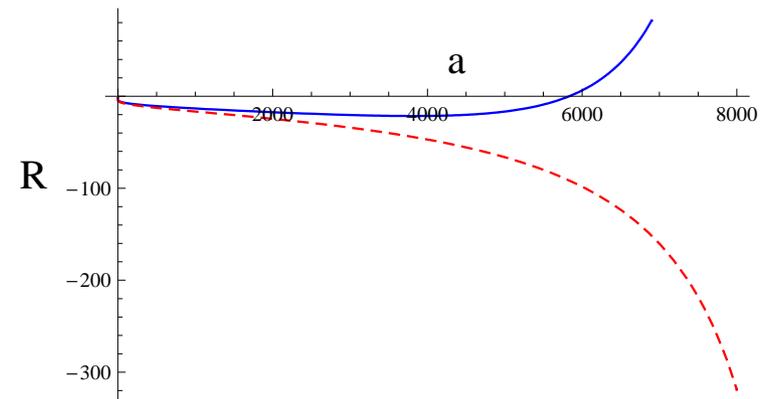
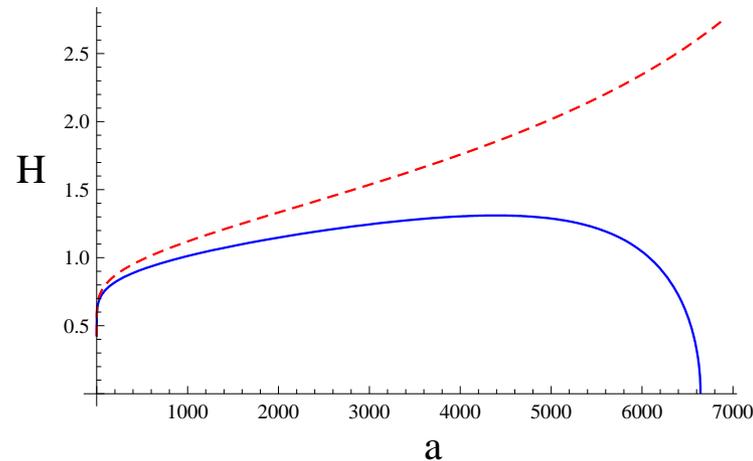
LQC resolves the initial (final) big bang (crunch) singularity. However, the curvature grows unbounded both in classical theory and LQC when the sudden singularity is approached.

Quantum geometry does not control divergence of curvature. A generic feature of Type II models in LQC. (Generalization of conclusion by Cailleteau, Cardoso, Vandersloot & Wands (2008)).

Type III Singularity and its Resolution

Parameter: $\alpha > 1$.

In the classical theory energy density, pressure, Hubble, Ricci, ... diverge as $a \rightarrow a_o$ in finite time.

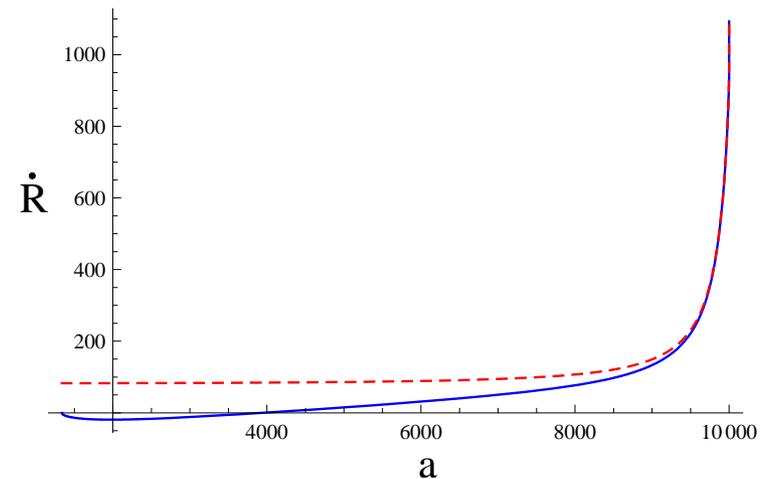
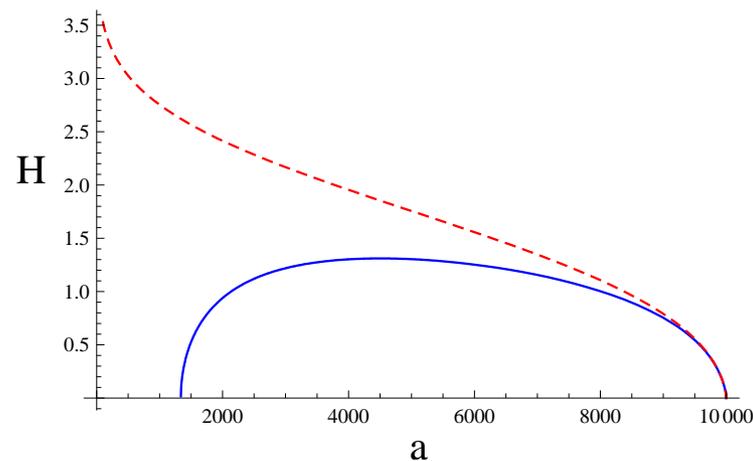


All curvature invariants and Hubble are bounded in LQC.

Type III singularity is resolved.

Type IV Singularity and its Non-Resolution

Parameter: $0 < \alpha < 1/2$. Given a value, it determines the order of derivative of R which diverges.



The Hubble and Ricci curvature are bounded and finite at the Type IV extremal event in both classical theory and LQC. However, derivative of R diverges. All curvature invariants are finite.

LQC does not resolve this curvature derivative singularity.

When is a singularity really a singularity?

Nature of Singularities

- **Hawking, Penrose, Geroch** Incompleteness Theorems: Existence of geodesics which can not be continued to arbitrary values of the affine parameter.
“A spacetime is singularity free if it is geodesically complete.” *Sagredo*^a

- What happens at or near the singularity? What does an observer experience?

Ellis & Schmidt: A singularity is strong if in falling objects or observer (along with his/her apparatus) experience infinite tidal forces and are thus destroyed. Else the singularity is weak. Strong detectors will survive.

- **Tipler's** criteria: In the FRW case, singularity is strong if the volume vanishes (or diverges). The following integral diverges as $\tau \rightarrow \tau_e$:

$$\int_0^\tau d\tau' \int_0^{\tau'} d\tau'' R_{ab} u^a u^b$$

- **Krolak's** criteria: Weaker than that of Tipler. If $\tau \rightarrow \tau_e$ the singularity is strong if following integral diverges:

$$\int_0^\tau d\tau' R_{ab} u^a u^b$$

Results from a general analysis

- Type II and Type IV singularities are not real singularities, even classically (Fernandez-Jambrina, Lazkoz (2006)). The Hubble is bounded and singularity occurs at a finite scale factor. This implies these are weak *a la* Tipler & Krolak. Geodesically complete extremal events ($t'' = -Ht'^2, t'^2 \sim 1/a^2$). Analogs of shell crossing singularities.

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- LQC ignores harmless weak curvature singularities. **Quantum geometry does not seem to over kill**.

Does LQC act smart?

- LQC seems to act in a simple and efficient way. It does minimal required to resolve the singularities. **Why?**(Corichi, PS (2008))
- Tied to a careful quantization of the Hamiltonian constraint which leads to improved dynamics (APS papers, sLQC).

Compute the expansion congruence of the geodesics for cosmological observers

$$\theta = g^{ab} B_{ab}, \quad B_{ab} := \nabla_b \xi_a$$

It turns out that $\theta = K = \beta/\gamma$. And that is bounded in the improved quantization!

Consistency conditions and demanding correct IR and UV behavior picked out β uniquely from a very large class.(Corichi, PS (2008)). **In a large class of loop quantizations, (β, V) is the only choice which guarantees geodesic completeness.**

Harmonious convergence of various results.

Interesting results for the Bianchi and Black Hole interior spacetimes.

Summary

- Phenomenological models act as useful guides and initial probes to venture in to unknown regimes of quantum geometry. However, they should be used and interpreted with lot of care. They can teach us many lessons and revise some intuitions.
- The issue of (traditional) past (or future) singularity has been investigated in various models. Big Bang and Big Crunch are resolved. Bounce is generic in various situations (at an effective level).
- Earlier results on cyclic model based on ad-hoc suppression of anisotropies led to difficulties. The model turns viable in a more general treatment.
- Quantum geometry effects do not necessarily bind the curvature. There may exist physically interesting scenarios in which curvature diverges. However, these should be viewed as analogs of shell crossing singularities. Atleast for $k = 0$ model these are harmless extremal events and one can say that quantum geometry resolves all types of physical cosmological singularities.
- What are the lessons for more general cases? We should be careful in posing the singularity resolution question. Geodesic completeness is the key and curvature boundedness may be misleading in some situations. Problems with certain spacetimes (BH) may be resolvable.