# QFT in the expanding quantum spacetime

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# The background QFRW: kinematics I

- the homogeneous scalar field $T$, and its momentum $\pi_T$
  - the Hilbert space

$$\mathcal{H}_{\rm sc} \;=\; L^2(\mathbb{R}), \quad (\psi_1\,|\,\psi_2)_{\rm sc} \;:=\; \int_{\mathbb{R}} dT\overline{\psi_1}(T)\psi_2(T), \tag{1}$$

  - the operators

$$\hat{T}\psi(T) \;=\; T\psi(T), \qquad \hat{\pi}_T\psi(T) \;=\; \frac{\hbar}{i}\frac{d}{dT}\psi(T), \tag{2}$$

- the scale factor $a$ and the extrinsic curvature
  - the Hilbert space $\mathcal{H}_{\rm gr}$
  - the scale operator $\hat{a}$, the corresponding momentum $\hat{\pi}_a$ (or other operators instead)
- The total kinematical Hilbert space

$$\mathcal{H}_{\rm sc} \otimes \mathcal{H}_{\rm gr}$$

# The background QFRW: kinematics II

- More ingredients we need:
  - a 3-manifold $\Sigma$
  - a homogeneous and isotropic conformal geometry $[q^{(0)}]$
  - a compact, closed region $\Sigma_0 \subset \Sigma$
- $1 \otimes \hat{a}^3$ is the geometric volume of $\Sigma_0$
- $\hat{\pi}_T \otimes 1$ is the classical

$$\int_{\Sigma_0} \tilde{\pi}_T$$

- the quantum 3-metric tensor operator on $\Sigma$

$$\hat{q} \;=\; \hat{a}^2 q^{(0)} \;=\; \hat{a}^2 q^{(0)}_{ab} dx^a dx^b \tag{3}$$

where

$$\int_{\Sigma_0} d^3x \sqrt{\det q^{(0)}} \;=\; 1.$$

# The background QFRW: dynamics 1

- the quantum scalar constraint operator $\hat{C}$ defined in $\mathcal{H}_{\mathrm{sc}} \otimes \mathcal{H}_{\mathrm{gr}}$:

$$\hat{C} \;=\; \frac{1}{2}\hat{\pi}_T^2 \otimes \widehat{a^{-3}} \;+\; 1 \otimes \widehat{C_{\mathrm{gr}}}, \tag{4}$$

  where $\widehat{a^{-3}}$ and $\widehat{C_{\mathrm{gr}}}$ are suitable operators defined in $\mathcal{H}_{\mathrm{gr}}$.
- a physical quantum state $\Psi$: an evolution

$$\mathbb{R} \ni T \mapsto \Psi_T \in \mathcal{H}_{\mathrm{gr}} \tag{5}$$

  such that

$$\frac{\hbar}{i}\frac{d}{dT}\Psi_T \;=\; \hat{H}_{\mathrm{gr}}\,\Psi_T \quad , \tag{6}$$

  where

$$\hat{H}_{\mathrm{gr}} \;=\; \sqrt{-2\widehat{a^{-3}}^{\,-1}\widehat{C_{\mathrm{gr}}}}, \tag{7}$$

- $$(\Psi|\Psi')_{\mathrm{phys}} \;=\; (\Psi_T|\widehat{a^{-3}}\Psi'_T). \tag{8}$$

# The background QFRW: dynamics 2

- the physical quantum operators
  - the scalar field: $T$ becomes time, and

$$\hat{\pi}_T = \hat{H}_{\rm gr}$$

  - $\hat{a}$

$$\begin{aligned} T_0 &\mapsto \hat{a}^{\rm D}(T_0) \\ (\hat{a}^{\rm D}(T_0)\Psi)_T &= e^{i(T-T_0)\hat{H}_{\rm gr}}\hat{a}e^{-i(T-T_0)\hat{H}_{\rm gr}}\Psi_T \end{aligned} \quad (9)$$

    Or in other words, $\hat{a}^{\rm D}(T_0)\Psi$ is a solution such that at $T = T_0$ it takes the value $\hat{a}\Psi_{T_0}$.
  - the 3-metric tensor operator

$$\hat{q}^{\rm D}(T_0) := (\hat{a}^{\rm D}(T_0))^2\, q^{(0)}. \quad (10)$$

# The issue of the quantum 4-metric operator

Classically,

$$ds^2 = -N^2(t)dt^2 + a^2(t)q^{(0)}_{ab}dx^q dx^b,$$

The K-G equations

$$\frac{dT}{dt} = \frac{N}{a^3}\pi_T$$

Hence

$$N dt = \frac{a^3}{\pi_T} dT$$

and

$$ds^2 = -(\frac{a^3}{\pi_T})^2 dT^2 + a^2 q^{(0)}.$$

However, $\hat{a}$ and $\hat{\pi}_T = \hat{H}_{\rm gr}$ do not commute. Therefore, a choice of the ordering is needed in:

$$\widehat{ds^2} := -\vdots\hat{a}^6 \hat{H}_{\rm gr}^{-2}\vdots dT^2 + \hat{a}^2 q^{(0)}_{ab} dx^a dx^b. \qquad (11)$$

# *Probing the quantum geometry*

Remarkably, that non-uniqueness will not affect the quantum evolution of our test quantum fields along the quantum FRW spacetime. The evolution will be derived in the next section in an ambiguity free way and some ordering will emerge naturally. In this sense, probing the quantum FRW with the quantum test fields will bring in more physical information about the quantum spacetime geometry.

# QFT in curved spacetimes 1

Metric tensor

$$g \;=\; g_{\mu\nu} dx^\mu dx^\nu$$

Hyperbolic space-time $M$

- Scalar field

$$(\Box_g + m^2)\hat{\Phi} \;=\; 0 \tag{12}$$

$$\left[\hat{\Phi}(t,x), \hat{\Phi}(t',x')\right] \;=\; i(G^g_{\mathrm{adv}}(t,x,t',x') \;-\; G^g_{\mathrm{ret}}(t,x,t',x')) \tag{13}$$

$$\Phi^\dagger \;=\; \Phi. \tag{14}$$

- A quantum state $\langle\cdot\rangle$

$$\hat{\Phi}(t_1,x_1)...\hat{\Phi}(t_n,x_n) \;\mapsto\; \langle\hat{\Phi}(t_1,x_1)...\hat{\Phi}(t_n,x_n)\rangle \tag{15}$$

# QFT in curved spacetimes 2: algebraically

- $A$ – $\star$-algebra generated by the letters

$$\hat{\Phi}(f), \quad \text{where} \quad f \in \mathrm{C}_0^\infty(M) \tag{16}$$

modulo the relations:

$$\hat{\Phi}(\alpha f_1 + f_2) \;=\; \alpha\hat{\Phi}(f_1) + \hat{\Phi}(f_2) \tag{17}$$

$$\Phi((\Box + m^2)f) \;=\; 0, \tag{18}$$

$$\left[\hat{\Phi}(f_1), \hat{\Phi}(f_2)\right] \;=\; i\int (G^g_{\mathrm{adv}}(t,x,t',x') - G^g_{\mathrm{ret}}(t,x,t',x'))f_1(t,x)f_2(t',x')dtdxdt' \tag{19}$$

$$\hat{\Phi}(f)^* \;=\; \hat{\Phi}(\bar{f}) \tag{20}$$

- A quantum state is a linear, positive functional $\langle\cdot\rangle \;:\; A \;\mapsto\; \mathbb{C}$.

# QFT in curved spacetimes: $\langle 0| \cdot |0\rangle$

- Ingredients: a family of ("modes")

$$(\Box + m^2)u_k \;=\; 0$$

labeled by $k \in K$ (endowed with a measure $dk$), such that

$$2\mathrm{Im}\int_K dk\; u_k(t,x)\bar{u}_k(t',x') \;=\; G^g_{\mathrm{adv}}(t,x,t',x') \;-\; G^g_{\mathrm{ret}}(t,x,t',x')$$

- The corresponding state ($\sum$ w.r.t. the partitions of $\{1,...,2n\}$ into pairs $(j_1,l_1),...,(j_n,l_n)$, $j_i > l_i$.):

$$\langle 0|\hat{\Phi}(t,x)\hat{\Phi}(t',x')|0\rangle \;:=\; \int_K dk\; u_k(t,x)\bar{u}_k(t',x')$$

$$\langle 0|\hat{\Phi}(t_1,x_1)...\hat{\Phi}(t_{2n},x_{2n})|0\rangle \;:=\; \sum \prod_{i=1}^{n} \langle 0|\hat{\Phi}(t_{j_i},x_{j_i})\hat{\Phi}(t_{l_i},x_{l_i})|0\rangle$$

# QFT in FRW: a formulation compatible with LQC

- The background spacetime: $M = \mathbb{R} \times S^1 \times S^1 \times S^1$

$$ds^2 = -N^2(t)dt^2 + a^2(t)(dx^1dx^1 + dx^2dx^2 + dx^3dx^3)$$

- a family of quantum systems ("modes") labeled by $\vec{k} \in 2\pi\mathbb{Z}^3$:
  - the Hilbert space $\mathcal{H}_{\vec{k}} = L^2(\mathbb{R})$
  - operators: $\hat{q}_{\vec{k}}\psi(q_{\vec{k}}) = q_{\vec{k}}\psi(q_{\vec{k}}), \qquad \hat{p}_{\vec{k}}\psi(q_{\vec{k}}) = \frac{\hbar}{i}\frac{d}{dq_{\vec{k}}}\psi(q_{\vec{k}})$
  - Hamiltonian $\hat{H}_{\vec{k}}(t) = \frac{N(t)}{2a^3(t)}\{\hat{p}_{\vec{k}}^2 + (a^4(t)k^2 + a^6(t)m^2)\hat{q}_{\vec{k}}^2\}$
  - solutions to $\frac{d}{dt}\Psi_t = -\frac{i}{\hbar}\hat{H}_{\vec{k}}(t)\Psi_t$
- in our case
  - $t = T, \quad N(T) = \frac{a^3(T)}{\pi_T},$
  - $\hat{H}_{\vec{k}}(T) = \frac{1}{2\pi_T}\{\hat{p}_{\vec{k}}^2 + (a^4(T)k^2 + a^6(T)m^2)\hat{q}_{\vec{k}}^2\}$
- $\hat{\phi}_{\vec{k}}(T,x,y,z) = (\hat{q}^{\rm H}_{\vec{k}}(T) + i\hat{q}^{\rm H}_{-\vec{k}}(T))\exp i\vec{k}\vec{x} + {\rm c.c}$

# QFT in QFRW: the idea 1

- Our goal is the generalization of the quantum mode structure to the quantum spacetime case. Naively, it is just $a \mapsto \hat{a}$. However, it is not that simple and one has to go beyond those heuristic replacements.
- Expand the scalar constraint

$$C(N=1) \;=\; \int_{\Sigma_0} d^3x(\tilde{C}_{\mathrm{gr}}(q_{ab},\pi_q^{ab}) + \tilde{C}_{\mathrm{KG}}(T,\pi_T) + \tilde{C}_{\mathrm{KG}}(\phi,\pi_\phi))$$

- Around homogeneous isotropic data:
  - gravitational part: $q_{ab} \;=\; a^2 q_{ab}^{(0)}$ and $\pi_q^{ab} = \pi_a q_{ab}^{(0)}$ ,
  - the first KG: homogeneous $T, \pi_T$
  - the second KG: $\phi, \pi_\phi = 0$.
- Perturbations $\delta f$ of the trace part of the metric, and of the field $T$ and $\pi_T$, are assumed to satisfy

$$\int_{\Sigma_0} \sqrt{\det q^{(0)}}\delta f \;=\; 0.$$

# QFT in QFRW: the idea 2

$$C(1) \;=C^{(0)}(a,\pi_a,T,\pi_T)\;+\;\sum_{\vec{k}\in 2\pi\mathbb{Z}^3} C^{(2)}_{\vec{k}}(a,\delta\phi_{\vec{k}},\delta\pi_{\vec{k}})+$$

$$+\;C^{(2)}_{\rm other}(\delta q_{ab},\delta\pi^{ab}_q,\delta T,\delta\pi_T,)\;+\;C^{(3)}+... \tag{21}$$

where $C^{(0)}$ is the constraint evaluated at the homogeneous isotropic data with $N=1$, and

$$C^{(2)}_{\vec{k}}\;=\;\frac{1}{2}\{\frac{p^2_{\vec{k}}}{a^3}+(a^3m^2+ak^2)q^2_{\vec{k}}\} \tag{22}$$

where we drop $\delta$

$$q_{\vec{k}}\;\equiv\;\delta q_{\vec{k}},\qquad p_{\vec{k}}\;\equiv\delta p_{\vec{k}}$$

# QFT in QFRW: the idea 3

- In the classical theory, up to the first order
  - the constraint $C^{(0)} = 0$
  - the dynamics of each of the modes $q_{\vec{k}}, p_{\vec{k}}$:

$$\frac{dq_{\vec{k}}}{dt} = \{q_{\vec{k}}, C^{(2)}_{\vec{k}}\}, \quad \frac{dp_{\vec{k}}}{dt} = \{p_{\vec{k}}, C^{(2)}_{\vec{k}}\}$$

- In the quantum theory however, the dynamics of the quantum FRW spacetime coupled with the field $T$ as well as the time itself, emerge in the space of solutions to the quantum scalar constraint. Therefore, to define the dynamics of each quantum mode $\phi_{\vec{k}}$ interacting with the quantum background spacetime *consistently*, we will consider in the next subsection a quantum theory given by the following part of the scalar constraint operator

$$C^{(0)} + C^{(2)}_{\vec{k}}. \tag{23}$$

We can ignore the other terms is that approximation, as well as the scalar constraints given by none-constant $\partial N$, because different perturbations do not interact with each other.

# A quantum KG mode in QFRW 1

- The kinematical Hilbert space $\mathcal{H} \ = \ \mathcal{H}_{\rm sc} \otimes \mathcal{H}_{\rm gr} \otimes \mathcal{H}_{\vec{k}}$
- The quantum fields:
  $\hat{T} \otimes 1 \otimes 1, \quad \hat{\pi_T} \otimes 1 \otimes 1, \quad 1 \otimes \hat{a} \otimes 1, \quad 1 \otimes 1 \otimes \hat{q}_{\vec{k}}, \quad 1 \otimes 1 \otimes \hat{p}_{\vec{k}}$
- The constraint operator

$$\hat{C} \ = \ \frac{1}{2}\hat{\pi}_T^2 \otimes \widehat{a^{-3}} \otimes 1 + 1 \otimes \widehat{C_{\rm gr}} \otimes 1 + \frac{1}{2} \otimes \widehat{a^{-3}} \otimes \hat{p}_{\vec{k}}^2 + \frac{1}{2} \otimes (k^2 \hat{a}^3 \widehat{a^{-2}} + m^2 \hat{a}^3) \otimes \hat{q}_{\vec{k}}^2,$$

- states:
  - $\mathbb{R} \ni T \mapsto \Psi_T \in \mathcal{H}_{\rm gr} \otimes \mathcal{H}_{\vec{k}}$
  - such that

$$\frac{\hbar}{i}\frac{d}{dT}\Psi_T \ = \ \sqrt{\hat{H}_{\rm gr}^2 \otimes 1 \ - \ 1 \otimes \hat{p}_{\vec{k}}^2 \ - \ (k^2 \hat{a}^4 + m^2 \hat{a}^6) \otimes \hat{q}_{\vec{k}}^2}\, \Psi_T,$$

  where the operator $\mathcal{H}_{\rm gr}$ is the gravitational Hamiltonian defined before.

# A quantum KG mode in QFRW 2

- applying the operator identity and ignoring "..."

$$(A+B)^{\frac{1}{2}} = A^{\frac{1}{4}}(1+A^{-\frac{1}{2}}BA^{-\frac{1}{2}})^{\frac{1}{2}}A^{\frac{1}{4}} = A^{\frac{1}{4}}(1+\frac{1}{2}A^{-\frac{1}{2}}BA^{-\frac{1}{2}} - ...)A^{\frac{1}{4}} \tag{24}$$

- we derive our generalized Schroedinger equation

$$\frac{d}{dT}\Psi_T = \frac{i}{\hbar}(\hat{H}_{\mathrm{gr}} \otimes 1 - \hat{H}_{\vec{k}})\Psi_T, \tag{25}$$

where

$$\hat{H}_{\vec{k}} = \frac{1}{2}\hat{H}_{\mathrm{gr}}^{-1} \otimes \hat{p}_{\vec{k}}^2 + \frac{1}{2}\hat{H}_{\mathrm{gr}}^{-\frac{1}{2}}(k^2\alpha(\hat{a}) + m^2\beta(\hat{a}))\hat{H}_{\mathrm{gr}}^{-\frac{1}{2}} \otimes \hat{q}_{\vec{k}}^2 \tag{26}$$

In the interaction picture

$$\frac{d}{dT}\Psi_T^{\mathrm{int}} \;=\; -\frac{i}{\hbar}\hat{H}_{\vec{k}}(T)\Psi_T^{\mathrm{int}} \tag{27}$$

where

$$\hat{H}_{\vec{k}}(T) \;=\;$$

$$\frac{1}{2}\hat{H}_{\mathrm{gr}}^{-1}\otimes\hat{p}_{\vec{k}}^2 \;+\; \frac{1}{2}\hat{H}_{\mathrm{gr}}^{-\frac{1}{2}}(k^2\alpha(\hat{a}(T)) \;+\; m^2\beta(\hat{a}(T)))\hat{H}_{\mathrm{gr}}^{-\frac{1}{2}}\otimes\hat{q}_{\vec{k}}^2 \tag{28}$$

and

$$\hat{a}(T) \;:=\; e^{-i(T-T_1)\hat{H}_{\mathrm{gr}}}\hat{a}e^{i(T-T_1)\hat{H}_{\mathrm{gr}}}, \tag{29}$$

and

$$\Psi_T \;=\; e^{i(T-T_1)\hat{H}_{\mathrm{gr}}}\Psi_T^{\mathrm{int}}. \tag{30}$$

# classical versus quantum background

$$-i\frac{d}{dT}\Psi_T = \hat{H}_{\vec{k}}(T)\Psi$$

$$\hat{H}_{\vec{k}}(T) = \begin{cases} \frac{1}{2\pi_T}\{\hat{p}^2_{\vec{k}} + (a^4(T)k^2 + a^6(T)m^2)\cdot\hat{q}^2_{\vec{k}}\} \\ \frac{1}{2}\hat{\pi}_T^{-\frac{1}{2}} \otimes 1 \circ \{1 \otimes \hat{p}^2_{\vec{k}} + (\hat{a}^4(T)k^2 + \hat{a}^6(T)m^2) \otimes \hat{q}^2_{\vec{k}}\} \circ \hat{\pi}_T^{-\frac{1}{2}} \otimes 1, \end{cases}$$

- $\Psi_T$
  - $\in \mathcal{H}_{\vec{k}}$ is a state of the quantum $\vec{k}$-mode
  - $\in \mathcal{H}_{\mathrm{gr}} \otimes \mathcal{H}_{\vec{k}}$ is a state of the quantum geometry and the $\vec{k}$-mode
- $\pi_T$
  - is a constant real number
  - - $= \hat{H}_{\mathrm{gr}}$, operator defined in $\mathcal{H}_{\mathrm{gr}}$,
    - $[\hat{H}_{\mathrm{gr}}, \hat{a}] \neq 0$
- $T \mapsto a(T)$
  - is a given real valued function,
  - is the given operator valued function
    $\hat{a}_T = \exp(-i(T-T_1)\hat{H}_{\mathrm{gr}})\,\hat{a}\,\exp(i(T-T_1)\hat{H}_{\mathrm{gr}})$,

# The comparizon in the Heisenberg picture

$$
\begin{aligned}
\frac{d}{dT}U(T) &= -i\hat{H}_{\vec{k}}(T)U(T) \\
\hat{A} \mapsto \hat{A}^{\mathrm{H}}(T) &= U(T)^{-1}\hat{A}U(T) \\
\frac{d}{dT}\hat{A}^{\mathrm{H}}(T) &= i[\hat{H}^{\mathrm{H}}_{\vec{k}}(T), \hat{A}^{\mathrm{H}}(T)]
\end{aligned}
$$

$$
\hat{H}^{\mathrm{H}}_{\vec{k}}(T) = \begin{cases}
\frac{1}{2\pi_T}\{\hat{p}^{\mathrm{H}\,2}_{\vec{k}}(T) & + (a^4(T)k^2 + a^6(T)m^2)\hat{q}^{\mathrm{H}}_{\vec{k}}(T)^2\} \\
\\
\frac{1}{2}\hat{\pi}_T^{\mathrm{H}\,-\frac{1}{2}}(T) \otimes 1 \circ & \\
\{\, 1 \otimes \hat{p}^{\mathrm{H}\,2}_{\vec{k}}(T) & + (\hat{a}^{\mathrm{H}\,4}(T)k^2 + \hat{a}^{\mathrm{H}\,6}(T)m^2) \otimes \hat{q}^{\mathrm{H}\,2}_{\vec{k}}(T)\,\} \\
 & \circ\hat{\pi}_T^{\mathrm{H}\,-\frac{1}{2}}(T) \otimes 1
\end{cases}
$$

The key difference is:

- $U(T)^{-1}a(T)U(T) \;=\; a(T)$
- $\hat{a}^{\mathrm{H}}(T) \;=\; U(T)^{-1}\hat{a}(T)U(T)$
- The advantage $[A^{\mathrm{H}}, B^{\mathrm{H}}] \;=\; [A, B]^{\mathrm{H}}$

# Admitting finitely many $\vec{k}$-modes

- $C^{(0)} \; + \; \sum_{i=1}^{n} C^{(2)}_{\vec{k}_i}$.
- states: $T \mapsto \Psi_T \in \mathcal{H}_{\rm gr} \otimes \mathcal{H}_{\vec{k}_1} \otimes ... \otimes \mathcal{H}_{\vec{k}_n}$.
- the interaction picture

$$\frac{d}{dT}\Psi^{\rm int}_T \;=\; -\frac{i}{\hbar}\sum_{i=1}^{n}\hat{H}_{\vec{k}_i}(T)\Psi^{\rm int}_T, \tag{31}$$

- a price: the modes do know about each other

# The semiclassical quantum geometry

The semiclassical approximation:

$$\Psi_T^{\rm int} \;=\; \Psi_{{\rm gr},0} \otimes \Psi_{\vec{k},T}$$

$$\Psi_{{\rm gr},0} \in \mathcal{H}_{\rm gr}$$

$$\langle \cdot \rangle \;=\; (\Psi_{{\rm gr},0} | \,\cdot\, \Psi_{{\rm gr},0})_{\rm gr}$$

$$\frac{d}{dt}\Psi_{\vec{k},T} \;=\; \langle \hat{H}_{\vec{k}} \rangle \Psi_{\vec{k},T} \tag{32}$$

$$\langle \hat{H}_{\vec{k}} \rangle \;=\; \frac{1}{2}(\langle \hat{\pi}_T^{-1} \rangle \hat{p}_{\vec{k}}^2 \;+\; (k^2 \langle \hat{\pi}_T^{-\frac{1}{2}} \hat{a}(T)^4 \hat{\pi}_T^{-\frac{1}{2}} \rangle + m^2 \langle \hat{\pi}_T^{-\frac{1}{2}} \hat{a}(T)^6 \hat{\pi}_T^{-\frac{1}{2}} \rangle) \hat{q}_{\vec{k}}^2) \tag{33}$$

# *Emergence of the classical geometry?*

Is there a classical *geometry* which gives the same result? Recall:

$$ds^2 \ = \ -N^2(T)dT^2 + a(T)^2 q^{(0)},$$

$$\hat{H}_{\vec{k}}(T) \ = \ \frac{N(T)}{2a^3(T)}\{\hat{p}^2_{\vec{k}} + (a^4(T)k^2 + a^6(T)m^2)\hat{q}^2_{\vec{k}}\}, \qquad (34)$$

In the case of the massless mode, given: $\langle \hat{\pi}_T^{-1} \rangle \in \mathbb{R}$ and $T \mapsto \langle \hat{\pi}_T^{-\frac{1}{2}} \beta(\hat{a}(T)) \hat{\pi}_T^{-\frac{1}{2}} \rangle$ there is unique solution $T \mapsto a(T)$ and $T \mapsto N(T)$ such that

$$\frac{N(T)}{a^3(T)} \ = \ \langle \hat{\pi}_T^{-1} \rangle, \qquad N(T)a(T) \ = \ \langle \hat{\pi}_T^{-\frac{1}{2}} \hat{a}(T)^4 \hat{\pi}_T^{-\frac{1}{2}} \rangle. \qquad (35)$$

However, in the massive mode case, the existence condition is

$$\frac{\langle \hat{\pi}_T^{-\frac{1}{2}} \hat{a}(T)^4 \hat{\pi}_T^{-\frac{1}{2}} \rangle^3}{\langle \hat{\pi}_T^{-1} \rangle^3} \ = \ \frac{\langle \hat{\pi}_T^{-\frac{1}{2}} \hat{a}(T)^6 \hat{\pi}_T^{-\frac{1}{2}} \rangle^2}{\langle \hat{\pi}_T^{-1} \rangle^2}. \qquad (36)$$

# Open problems

- understanding the creation anihilation operators:

$$\hat{p}_{\vec{k}} \pm i\hat{a}^4 \hat{q}_{\vec{k}}$$

- time independent vacuum?
- accomodating infinitely many modes
- QFT in QFRW as the first step in perturbing the full theory aroung quantum background.
- improving the time definition in QFRW