Tensor modes in loop quantum cosmology

with G. Hossain, arXiv:0810.4330 [gr-qc]

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PENNSTATE 1855

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Aims of the talk

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- To compute cosmological observables from LQC linear tensor perturbations.

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- To discuss related issues and future directions.

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$$\rho_c \equiv \frac{3}{8\pi G\gamma^2\bar{\mu}^2 p} \propto a^{-2(1-2n)}.$$

$$\alpha = \frac{1+n}{3r}\lambda\left(\left|1+\frac{1}{\lambda}\right|^{\frac{3r}{2(1+n)}} - \left|1-\frac{1}{\lambda}\right|^{\frac{3r}{2(1+n)}}\right), \qquad \lambda \sim \mathcal{V}^{2(1+n)/3}$$

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where

$$c=4(1+n)\,,\qquad \alpha_c=\frac{[3r-4(1+n)][3r-2(1+n)]}{3^4 2}\left(\frac{\Delta_{\rm Pl}}{\Delta}\right)^2.$$

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Assuming $\Delta=\Delta_{\rm Pl}$

$$4<c\leq 6\,,\qquad -0.01\approx -\tfrac{1}{162}<\alpha_c<\tfrac{1}{9}\approx 0.1$$

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$$1 < q_\alpha < 6\,, \qquad 1.6 \approx \frac{3^{3/4}}{\sqrt{2}} < \alpha_q < \frac{27}{4} \approx 6.8$$

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and

$$\{\delta K^i_a(\mathbf{x}), \delta E^b_j(\mathbf{y})\} = 8\pi G\delta^b_a\delta^i_j\delta(\mathbf{x}, \mathbf{y})$$

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We solve it in large- and small-volume regimes separately.

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Inflation occurs for $p < -1$ (de Sitter: $p = -1$), superinflation when $-1 < p < 0$.

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- Large- and short-wavelength limits of the solution ($\nu > 0$)

$$
\begin{aligned}
w_k &\sim -iC_1 \frac{2^\nu \Gamma(\nu)}{\pi} (-kz)^{1/2-\nu}\,, \qquad |kz| \ll 1\,, \\
w_k &\sim C_1 \sqrt{\frac{2}{\pi}} e^{-i\left(kz + \frac{\pi}{2}\nu + \frac{\pi}{4}\right)}\,, \qquad |kz| \gg 1\,.
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Operator $\hat{u}_k = a\hat{h}_k = w_k a_k + w_k^* a_k^\dagger$ obeys

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Plugging in the short-scale solution, one gets $|C_1| = \sqrt{8\pi^2\ell_{\rm Pl}^2/k}$.

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Tensor spectrum:

$$A_T^2 \equiv \frac{\mathcal{P}_h}{100} \equiv \frac{k^3}{200\pi^2 a^2} \sum_{+,\times} \left\langle |\hat{u}_{k\ll\mathcal{H}}|^2 \right\rangle \Big|_{k=k_*}$$

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Tensor spectral index:

$$n_T \equiv \frac{d\ln A_T^2}{d\ln k}\Big|_{k=k_*} = \frac{2(\epsilon+q_\alpha)}{\epsilon+q_\alpha-1}$$

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- If $r \sim 10^{-8}$, still these bounds are $n_T < 1$.

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$$\partial_\tau^2 w_k + c\mathcal{H}(\alpha-1)\partial_\tau w_k + \{(2\alpha-1)k^2 + \mathcal{H}^2[\epsilon - 2 - c(\alpha-1)]\}w_k \approx 0.$$

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- Correction term decays in time.

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Tensor index:

$$n_T \approx 2(1+p+c p\delta_{\rm Pl}) = \frac{-2(\epsilon + c\delta_{\rm Pl})}{1-\epsilon}\,.$$

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- Quasi-classical result reliable, but scalar sector still under inspection.