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Loop Quantum Cosmology holonomy corrections to inflationary models

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Phenomenology of loop corrections

- Introduction to cosmology and inflation
- Background evolution in the FRW cosmology with the holonomy loop correction
- Evolution of the scalar metric perturbations
- Effective comoving sonic horizon
- Power spectrum of the initial energy density perturbations

Let us consider the FRW Universe with k=0. From Einstein equations one obtains Friedmann equations

$$H^2 = \frac{\rho}{3} \qquad \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3P) \quad \Rightarrow \quad \dot{H} = -\frac{1}{2}(\rho + P)$$

To solve these equations we need to know the barotropic parameter ω, where P=ωρ, or we need to know the equation of motion for matter fields

Inflation appears when $\frac{\ddot{a}}{a} > 0$ and the most popular scenarios assume ω≈-1

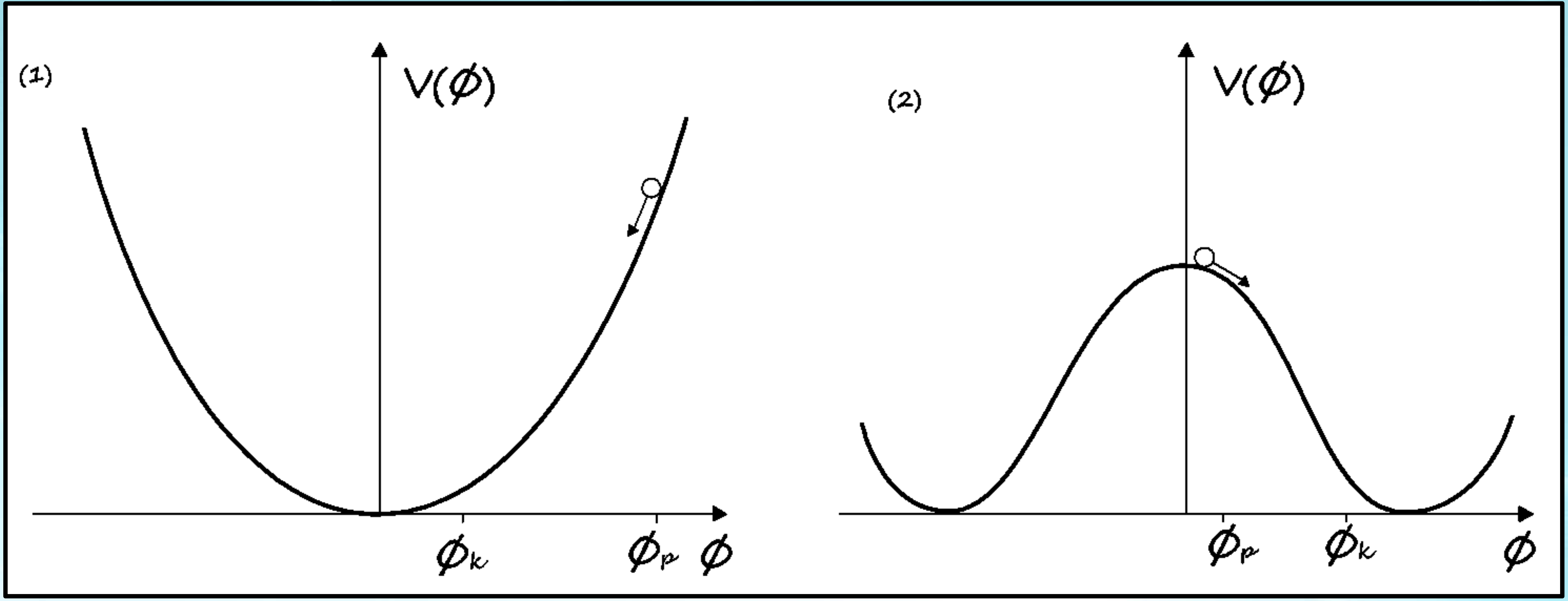
We need ρ≈-P ⇒ rather unnatural equation of state!

Then $a(t) \approx e^{Ht}$ and the comoving Hubble horizon $\frac{1}{aH}$ decreases rapidly

The most popular model: The early Universe is dominated by the scalar field ϕ

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \qquad P = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \Rightarrow$$ to get ω≈-1 we need $\dot{\varphi}^2 << V(\varphi)$

$$\ddot{\varphi} + 3H\dot{\varphi} + V' = 0$$ ⇒ Equation of motion for the inflaton



Now we work with the inflaton field ϕ(t) and its perturbation δϕ(x,t)

$$\rho(t) \rightarrow \rho^{(o)}(t) + \delta\rho(\vec{x},t) \qquad g_{oo}(t) \rightarrow g_{oo}(t) + \delta g_{oo}(\vec{x},t) = -(1 + 2\Phi(\vec{x},t))$$

$$P(t) \rightarrow P^{(o)}(t) + \delta P(\vec{x},t) \qquad g_{ij}(t) \rightarrow g_{ij}(t) + \delta g_{ij}(\vec{x},t) = a^2(1 - 2\Psi(\vec{x},t))\delta_{ij}$$

We do not have any anisotropic pressure $\Rightarrow \Phi = \Psi$

$$\delta G_{\mu\nu} = \delta T_{\mu\nu} \Rightarrow$$

$$3H\dot{\Psi} + 3H^2\Psi + \frac{\Delta}{a^2}\Psi = -\frac{1}{2}\delta\rho \qquad \ddot{\Psi} + 4H\dot{\Psi} + \left(3H^2 + 2\dot{H}\right)\Psi = \frac{1}{2}\delta P$$

After Fourier trans-
formation we have

$$\ddot{\Psi} + (4 + 3c_s^2)H\dot{\Psi} + [2\dot{H} + 3H^2(1 + c_s^2) - k^2 c_s^2/a^2]\Psi = 0$$

For $kc_s \ll aH$ we have the solution $\Psi \approx const$ ⟶ Perturbations are frozen outside the comoving sonic horizon

For the strong slow-roll approximation
δϕ evolves like the masless scalar field

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} + k^2\delta\varphi/a^2 = 0$$

Conformal time: a(η)dη=dt ⇒ For $a(t) \approx e^{Ht}$ we obtain $\eta = -\frac{1}{aH}$

From the equation of motion for δϕ(k,η) we get

$$\delta\varphi = e^{-ik\eta}\frac{\eta H}{\sqrt{2k}}\left(i/k\eta - 1\right)$$

Then we define the Power spectrum of δϕ by

$$\mathcal{P}_{\delta\varphi}(k,\eta) = \frac{k^3}{2\pi^2}\left|\delta\varphi(k,\eta)\right|^2$$

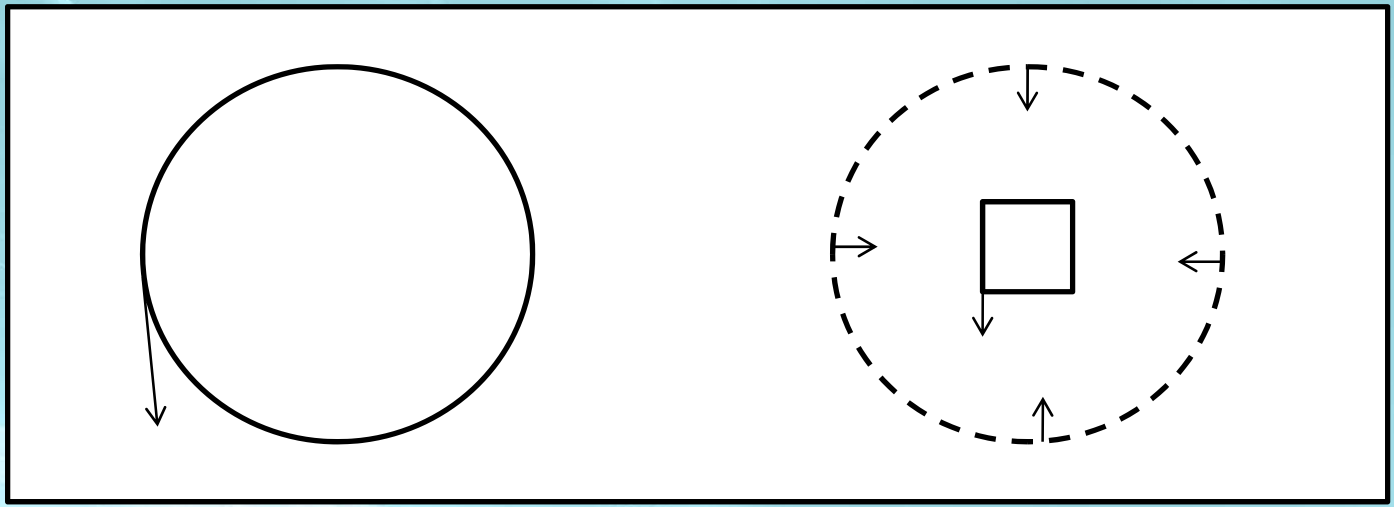
And finally… $\mathcal{P}_{\delta\varphi}(k,\eta) = \frac{1}{4\pi^2}H^2\left(k^2\eta^2+1\right)$ ⇒ we can not measure that!

$\mathcal{R} = \Psi + \frac{H}{\dot{\varphi}}\delta\varphi$ ⟵ Mukhanov-Sasaki variable. This is something like $\frac{\delta\rho}{\rho}$

In slow-roll approximation $\mathcal{P}_{\mathcal{R}} \approx \frac{H^4}{4\pi^2\dot{\varphi}^2} \approx \frac{V}{24\pi^2\varepsilon}$

Ashtekar variables in the FRW Universe: $c = \gamma \dot{a}$ $p = a^2$ $\{c, p\} = \frac{\gamma}{3}$

The hamiltonian $H = -\frac{3N}{\gamma^2}\sqrt{|p|}\,c^2 + H_{mat}$ gives us Friedmann equations



The parallel transport around the loop changes the vector. If we would shrink the loop to the smallest possible size we would get the elementary correction

By considering the loop quantum gravity modifications to the c we get

$$c \to \sin(c l_j/\sqrt{p}) \frac{\sqrt{p}}{l_j}$$

, where l_j is the quantum of length.

The holonomy loop correction does not changes p!

$$l_j \propto l_{pl}(j(j+1))^{1/4}$$

This is an extremely important variable!

No specific value of j chosen by nature! $j = \frac{1}{2}, 1, \frac{3}{2}, \ldots$

$$\rho_{cr} = \frac{3}{\gamma^2 l_j^2} \propto \frac{M_{pl}^4}{\sqrt{j(j+1)}}$$

Critical (maximal) energy density of the Universe

For $8\pi G = 1$ and big values of j we have $\rho_{cr} \sim 1/j$

Friedmann equations

$$H^2 = \frac{\rho}{3}\left(1 - \frac{\rho}{\rho_{cr}}\right)$$

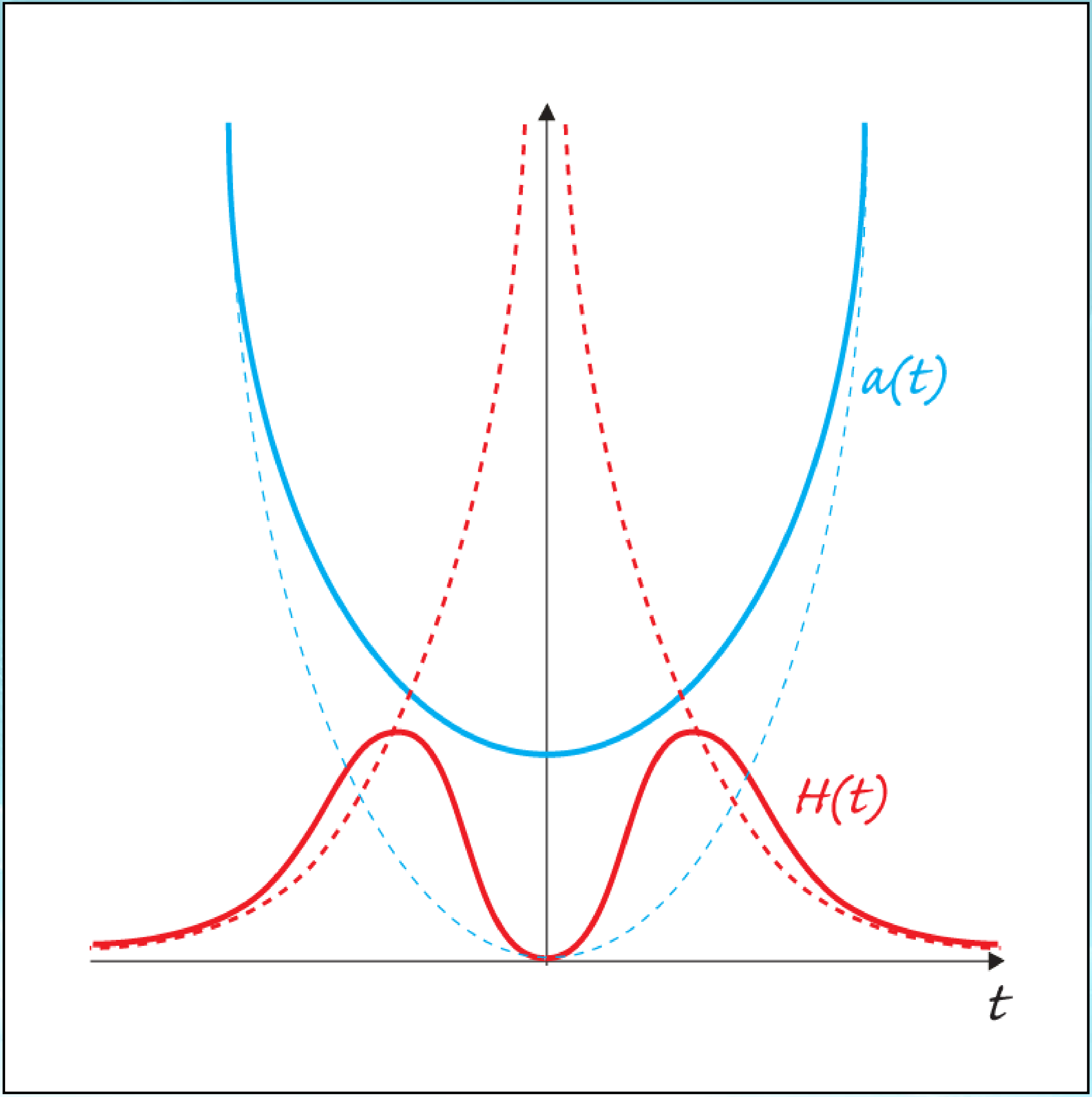
$$\dot{H} = -\frac{1}{2}(\rho + P)\left(1 - 2\frac{\rho}{\rho_{cr}}\right)$$

$\rho/\rho_{cr} \rightarrow 0 \Rightarrow$ normal FRW

Effective variables

$$\frac{\rho_{eff}}{3} = \frac{\rho}{3}\left(1 - \frac{\rho}{\rho_{cr}}\right)$$

$$P_{eff} = P\left(1 - 2\frac{\rho}{\rho_{cr}}\right) - \frac{\rho^2}{\rho_{cr}}$$



We can write Hamilton equations for $ds^2 = -N^2 dt^2 + p(d\vec{x})^2$, where

$$p = a^2(1 - 2\Psi) \qquad N = (1 + \Phi)$$

We consider k $\rightarrow$ 0 so perturbations are functions of time only. We do not have any anisotropic pressure $\Rightarrow \Phi = \Psi$ and from perturbated friedmann equations we have

$$3H\dot{\Psi} + 3H^2\Psi = -\frac{1}{2}\delta\rho_{eff} \qquad \ddot{\Psi} + 4H\dot{\Psi} + \left(3H^2 + 2\dot{H}\right)\Psi = \frac{1}{2}\delta P_{eff}$$

For the adiabatic perturbations we obtain

$$\ddot{\Psi} + (4 + 3c_{s_{eff}}^2)H\dot{\Psi} + [2\dot{H} + 3H^2(1 + c_{s_{eff}}^2)]\Psi = 0$$

This equation is almoust identical with the one from the standard FRW.
Perturbations are frozen outside the effective sonic horizon.

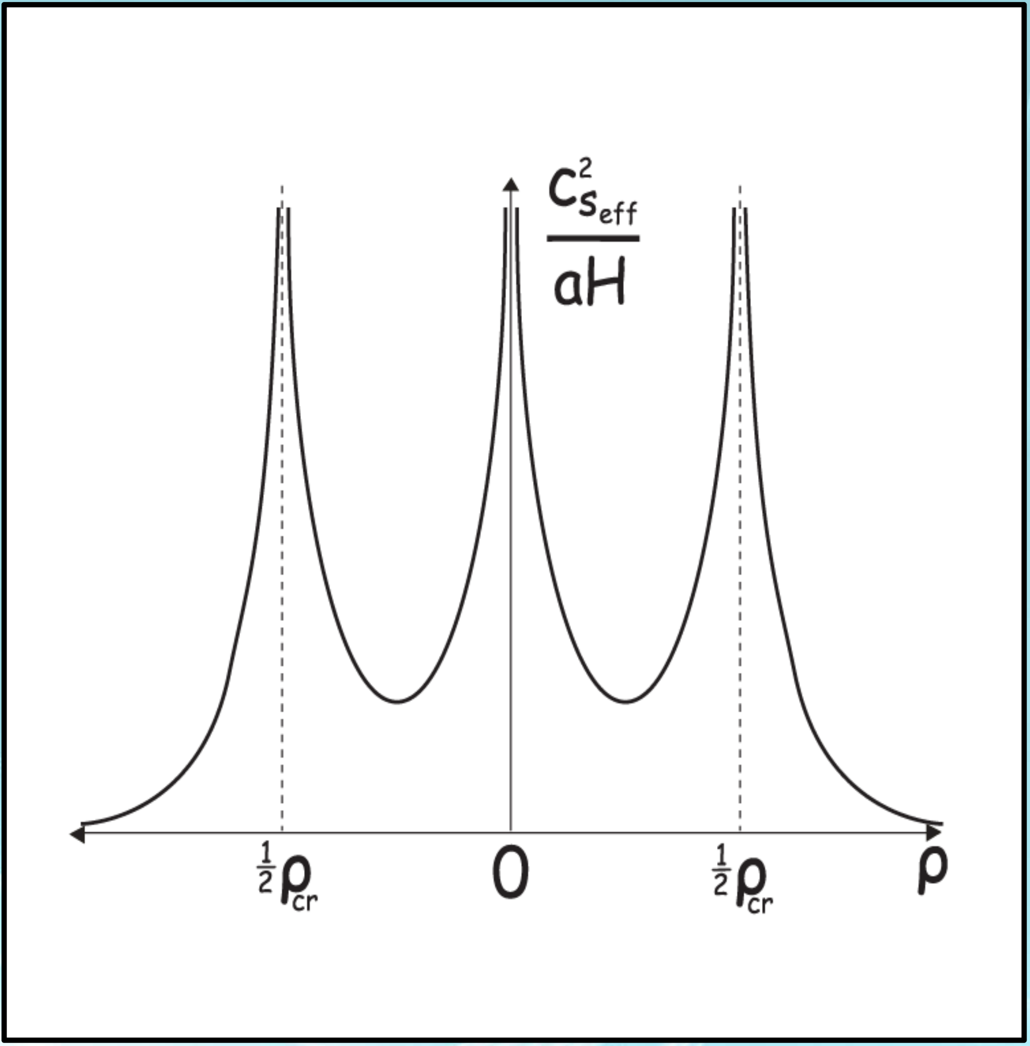
In LQC the effective speed of sound becomes infinite for $\rho = \rho_{cr}/2$

$$c^2_{s_{eff}} = \frac{\delta P_{eff}}{\delta \rho_{eff}} = c^2_s - 2\frac{(\rho + P)/\rho_{cr}}{1 - 2\rho/\rho_{cr}}$$

No conserved information left over from the $\dot{H} > 0$ period

The effective Big Bang scenario!

$c^2_{s_{eff}}$ is not a physical velocity!!!



Equations for the inflaton and it's perturbation are not changed by the loop correction

$$\delta\ddot{\varphi}+3H\delta\dot{\varphi}+k^2\delta\varphi/a^2=0$$

where $\delta\varphi$ is the inflaton perturbation

For the slow-roll approximation $\varepsilon=\frac{1}{2(1-\rho/\rho_{cr})}\frac{V'^2}{V^2}$, $V_{eff}=V(1-V/\rho_{cr})$

The power spectrum of the curvature perturbations is then in form of

$$\mathcal{P}_{\mathcal{R}_{loop}}=\frac{V_{eff}}{72\pi^2\epsilon}=(1-\frac{V}{\rho_{cr}})^2\mathcal{P}_{\mathcal{R}}$$

From the COBE normalisation we know, that if we want to avoid fine tuning we need to have $\rho_{cr}>(10^{16}\ \mathrm{GeV})^4$

This gives us the limit for j. For big values of j $\rho_{cr}\propto M_{pl}^4/j\Rightarrow j<10^{12}$

The spectral index $n_s(k)-1=2\eta-6\varepsilon$ is changed by the LQC to

$$n_{s_{loop}}(k)-1=\frac{n_s(k)-1}{(1-V/\rho_{cr})}$$

•FRW Universe in the low energy limit for LQC

•No information about the scalar perturbations can cross the ρ_{cr} energy regime untouched

•We can limit the quantum of length by the COBE normalisation

•More models fit's the data for strong LQC holonomy efects