Spikes in the Mixmaster regime of G_2 cosmologies

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Introduction: Singularity and BKL conjecture

How does an inhomogeneous cosmological model behave near a singularity?

Belinsky, Khalatnikov and Lifshitz (BKL) conjecture: model behavior is

1. vacuum dominated ("matter doesn't matter")
2. local (locally behaves like spatially homogeneous models)
3. oscillatory (called "Mixmaster" by Misner)

Will show that 2. is violated by spiky inhomogeneity.

Building blocks of Mixmaster oscillation

Building blocks made of vacuum, anisotropic, spatially homogeneous explicit solutions:
1. Kasner solution (Bianchi type I)
2. Taub solution (Bianchi type II)

Two Mixmaster prototypes:
1. Bianchi type IX
2. Bianchi type $VI^*_{-1/9}$

Visualizing the Mixmaster dynamics

Apply 3+1 decomposition (get intrinsic and extrinsic curvatures).

Intrinsic curvature describes the spatial curvature $^3R_{ab}$.

Extrinsic curvature describes the anisotropy of the expansion.

Factor out the overall expansion so that variables do not blow up at singularity.

Left with spatial curvature components N_{ab}

and expansion shear Σ_{ab}.

Building blocks of Mixmaster oscillation

Building blocks made of vacuum, anisotropic, spatially homogeneous explicit solutions:

1. Kasner solution (Bianchi type I) ($N_{ab} = 0$)
2. Taub solution (Bianchi type II) (N_{ab} has 1 non-zero eigenvalue)

Two prototypes:

1. Bianchi type IX (N_{ab} eigenvectors do not rotate, 3 non-zero eigenvalues)
2. Bianchi type VI$^*_{-1/9}$ (N_{ab} eigenvectors rotate, 2 non-zero eigenvalues)

Kasner solution

$$\Sigma_{ab} = \begin{pmatrix} -2\Sigma_+ & 0 & 0 \\ 0 & \Sigma_+ + \sqrt{3}\Sigma_- & 0 \\ 0 & 0 & \Sigma_+ - \sqrt{3}\Sigma_- \end{pmatrix}, \qquad \Sigma_+^2 + \Sigma_-^2 = 1.$$

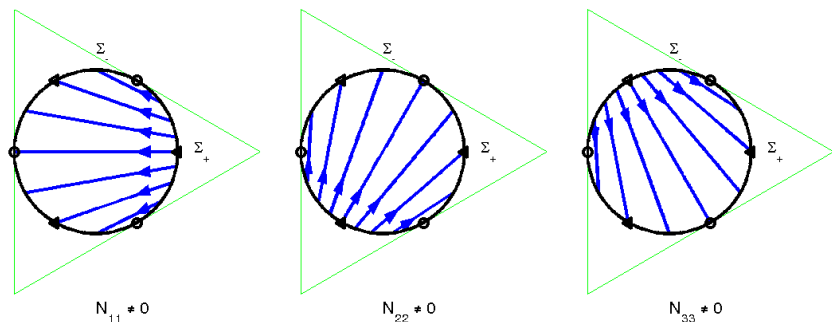Each Kasner solution is represented by a point on the unit circle.

Taub solution

$$\Sigma_{ab} = \begin{pmatrix} -2\Sigma_+ & 0 & 0 \\ 0 & \Sigma_+ + \sqrt{3}\Sigma_- & 0 \\ 0 & 0 & \Sigma_+ - \sqrt{3}\Sigma_- \end{pmatrix}, N_{ab} = \begin{pmatrix} N_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Each Taub solution is represented by an orbit connecting 2 Kasner points.
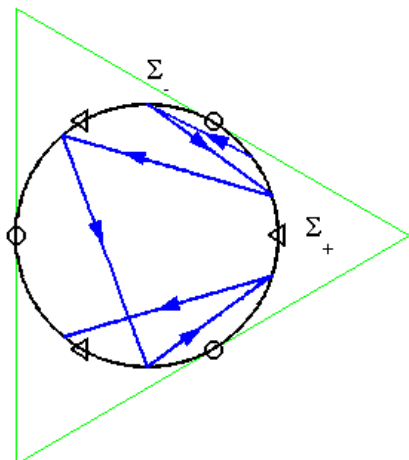
Kasner points and Taub orbits

3 orientations of Taub orbits projected onto the (Σ_+, Σ_-) plane. Arrows indicate evolution towards singularity.

Mixmaster orbit

Mixmaster orbit built from consecutive Taub orbits connecting the Kasner points.

Spikes discovered numerically

Beverly Berger & Vincent Moncrief 1993 found that spiky inhomogeneity develops near the singularity in numerical simulations of Gowdy models, which are inhomogeneous models with 2 KVFs (G_2) with special group action. Gowdy models have no Mixmaster dynamics.

Reason: $N_{ab}(t, x)$ eigenvalue may have a zero at certain locations. Taub solution is not possible here, and a spike forms as a result.

Do spikes also appear in general models with Mixmaster? If they do, how do they affect the Mixmaster dynamics?

Narrowing spikes make numerical simulations difficult. Luckily ...
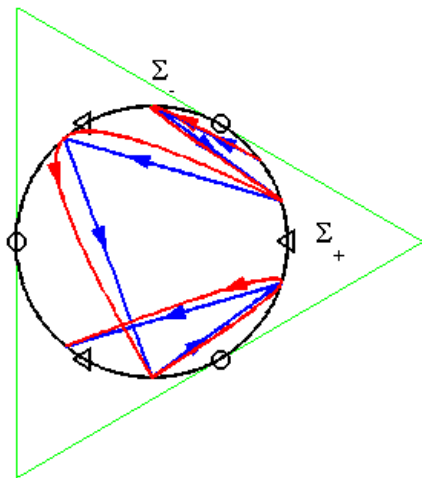
Explicit spike solution discovered

The explicit spike solution is found last year [WCL 2008].
Solution describes a spike that forms and then flattens out.

The spiky inhomogeneity in components N_- and $\Sigma_\times$
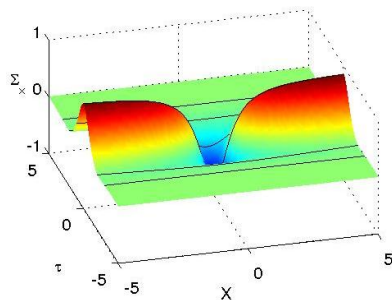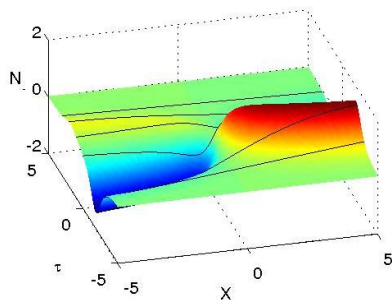
Spike orbits

One spike orbit is equivalent to two consecutive Taub orbits.

Numerical simulations of G_2 cosmologies

G_2 cosmologies have Mixmaster dynamics and spikes.
Numerical simulations of G_2 cosmologies are carried out
[WCL, Frans Pretorius, David Garfinkle and Lars Andersson 2008,
in preparation].

Simulations confirm that spikes
1. recur (repeatedly form and flatten out).
2. are an integral, inhomogeneous part of the Mixmaster dynamics.

Conclusion: New building blocks of Mixmaster oscillation

Building blocks made of vacuum, anisotropic explicit solutions.
1. Kasner solution (Bianchi type I)
2. Taub solution (Bianchi type II)
3. Spike solution (G_2 Gowdy class)

New Mixmaster prototype: G_2 inhomogeneous models.

Next step: Mixmaster dynamics in general inhomogeneous models.