Beyond Conformally-Flat Black-Hole Initial Data

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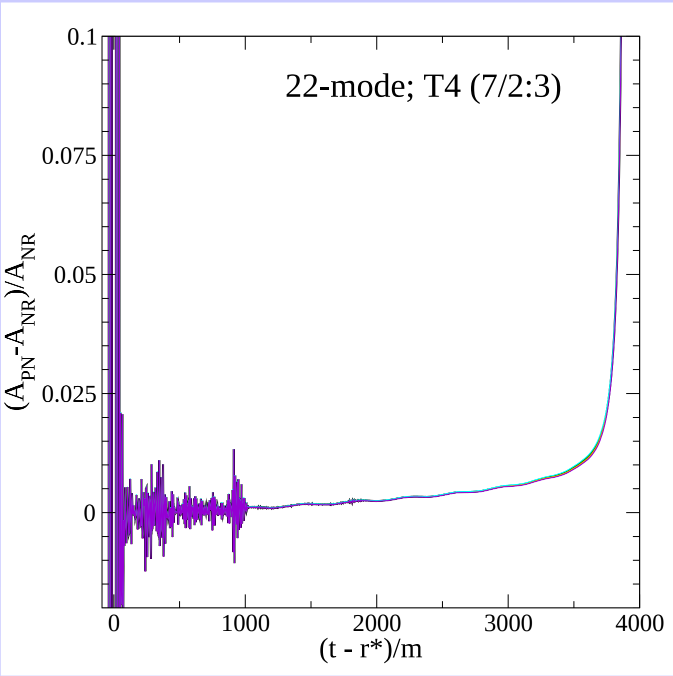
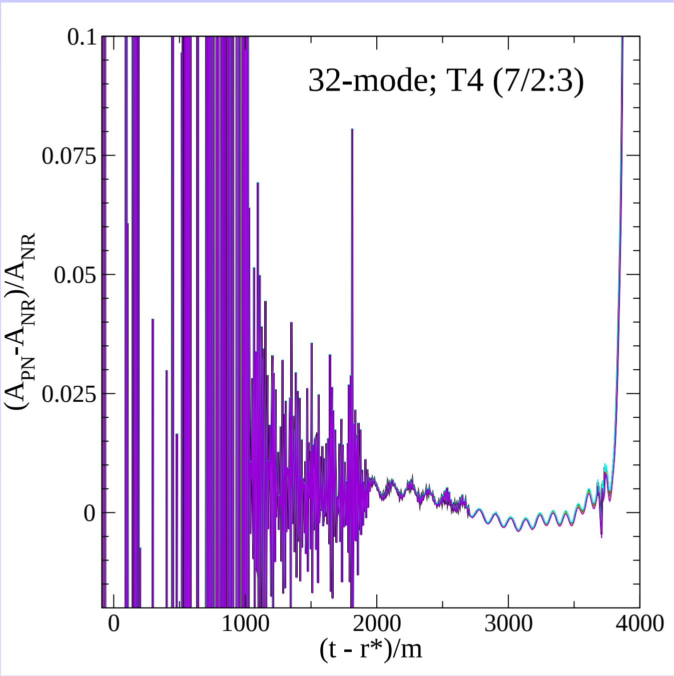
Overview

- Issues with Conformal Flatness
- Using the Tracefree part of $\partial_t K^{ij}$ to determine $\tilde{\gamma}_{ij}$
- A Reference Metric Approach for Initial Data
- Preliminary Tests

What We Typically Use/Conformal Flatness

- Most BBH evolutions use conformally flat initial data.
- CFID are the most readily available ID but we know conformal flatness implies junk radiation.
- Junk radiation doesn't affect the "gross physics" but it does cause problems with precision comparisons to PN inspiral and may eventually lead to problems with parameter estimation.





Comparison of BBH spectral evolution (Cornell/Caltech collab.) to PN.

Getting more out of $\partial_t K^{ij}$

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^j dt)(dx^j + \beta^i dt)$$

$$R^2 + K^2 - K_{ij}K^{ij} = 0$$

$$\nabla_j \left(K^{ij} - \gamma^{ij}K\right) = 0$$

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + 2\nabla_{(i}\beta_{j)}$$

$$\begin{aligned}\partial_t K_{ij} = -\nabla_i \nabla_j \alpha + \alpha \left[R_{ij} - 2K_{il}K^l_j + K K_{ij}\right] \\ +\beta^l \nabla_l K_{ij} + 2K_{l(i}\nabla_{j)}\beta^l\end{aligned}$$

Getting more out of $\partial_t K^{ij}$

$$\partial_t K = -\nabla^2\alpha + \alpha A_{ij}A^{ij} + \frac{1}{3}\alpha K^2 + \beta + \nabla_k K$$

$$\begin{aligned}\partial_t A_{ij} = &-(\nabla_i\nabla_j - \frac{1}{3}\nabla^2)\alpha + \beta^k\nabla_k A_{ij} + 2A_{k(i}\nabla_{j)}\beta^k \\ &+\alpha\left[R_{ij} - \frac{1}{3}\gamma_{ij}R - 2A_{ik}A^k_j + \frac{1}{3}KA_{ij}\right]\end{aligned}$$

An elliptic equation for the conformal metric was developed by Shibata, Friedman and Uryu [1] and implemented by Uryu, Limousin, Friedman, Gourgoulhon and Shibata [2].

A Reference Metric Approach

$$\delta\tilde{\Gamma}^k_{ij} \equiv \tilde{\Gamma}^k_{ij} - {}_f\Gamma^k_{ij} = \frac{1}{2}\tilde{\gamma}^{kl}\left[\, {}_f\nabla_i\tilde{\gamma}_{jl} + {}_f\nabla_j\tilde{\gamma}_{il} - {}_f\nabla_l\tilde{\gamma}_{ij}\right]$$

$$V^k \equiv \tilde{\gamma}^{ij}\delta\Gamma^k_{ij} = -\frac{1}{\sqrt{\tilde{\gamma}}}\, {}_f\nabla_l(\sqrt{\tilde{\gamma}}\tilde{\gamma}^{kl}) = 0$$

$${}_f\Delta\tilde{\gamma}^{ij} + ... = 0$$

Greg Cook was inspired to apply this with a reference metric approach to BBHs with excised boundaries requiring him and Thomas Baumgarte to develop the proper boundary conditions, the details of which will be in an upcoming paper.

Numerical Methodology

This approach creates a set of coupled nonlinear equations for the variables ψ, β^i, α and $\tilde{\gamma}^{ij}$. Currently we decouple the equations into two sets where we solve for ψ, β^i and α with a given $\tilde{\gamma}^{ij}$ and then do the reverse. This can be done iteratively back and forth. For this we use Harald Pfeiffer's spectral elliptic solver.

- Have I coded this correctly?

- Will this method work at all?

Preliminary Issues to Examine

- Does ${}_f\Delta\tilde{\gamma}^{ij} + ... = 0 \rightarrow det\ \tilde{\gamma} = det\ f$?
- Does ${}_f\Delta\tilde{\gamma}^{ij} + ... = 0$ preserve gauge conditions?
- Ellipticity of ${}_f\Delta\tilde{\gamma}^{ij}$...

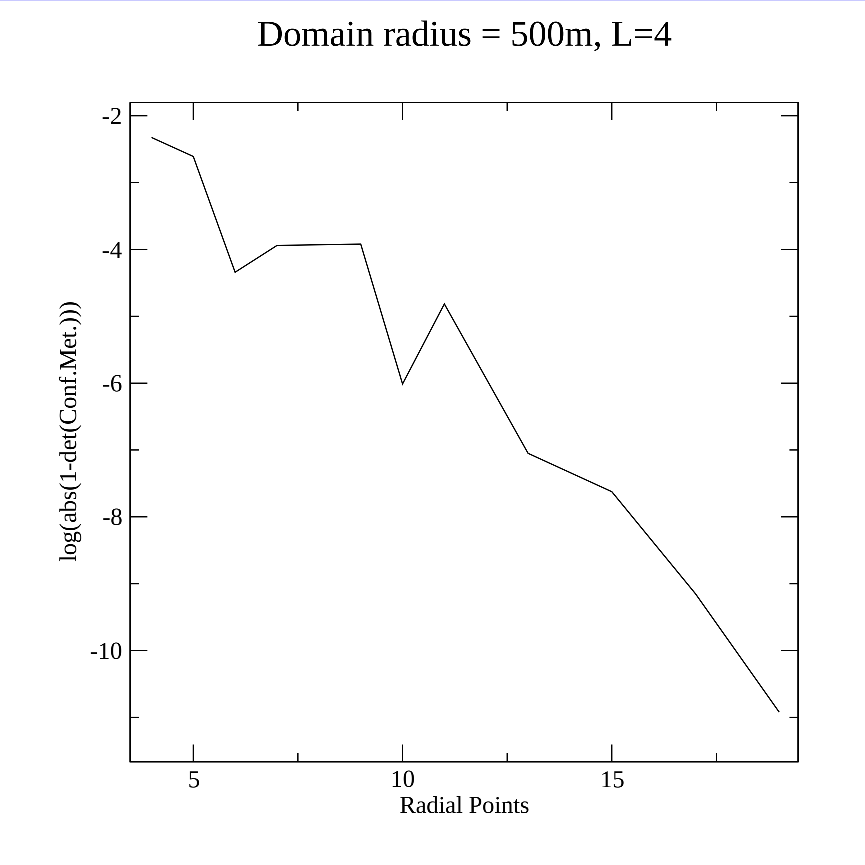
$$\left(\tilde{\gamma}^{kl} - \frac{\beta^k\beta^l}{\psi^4\alpha^2}\right)\ {}_f\nabla_k\ {}_f\nabla_l\tilde{\gamma}^{ij} + ...$$

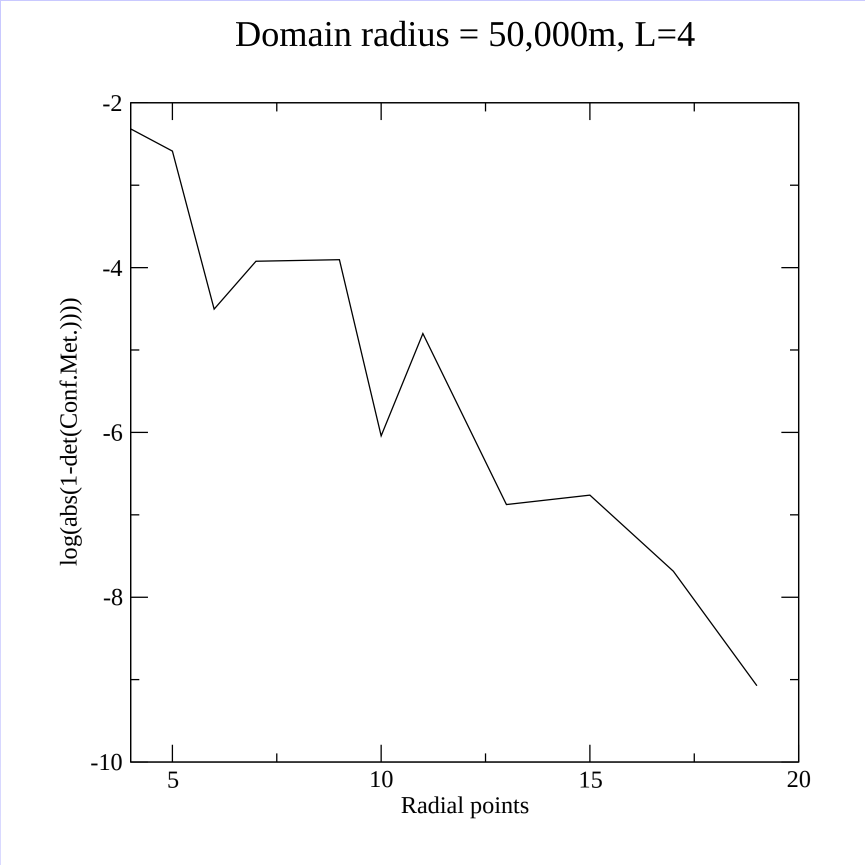
Tests

- Schwarzschild to test $det\ \tilde{\gamma} = det\ f$
- Kerr

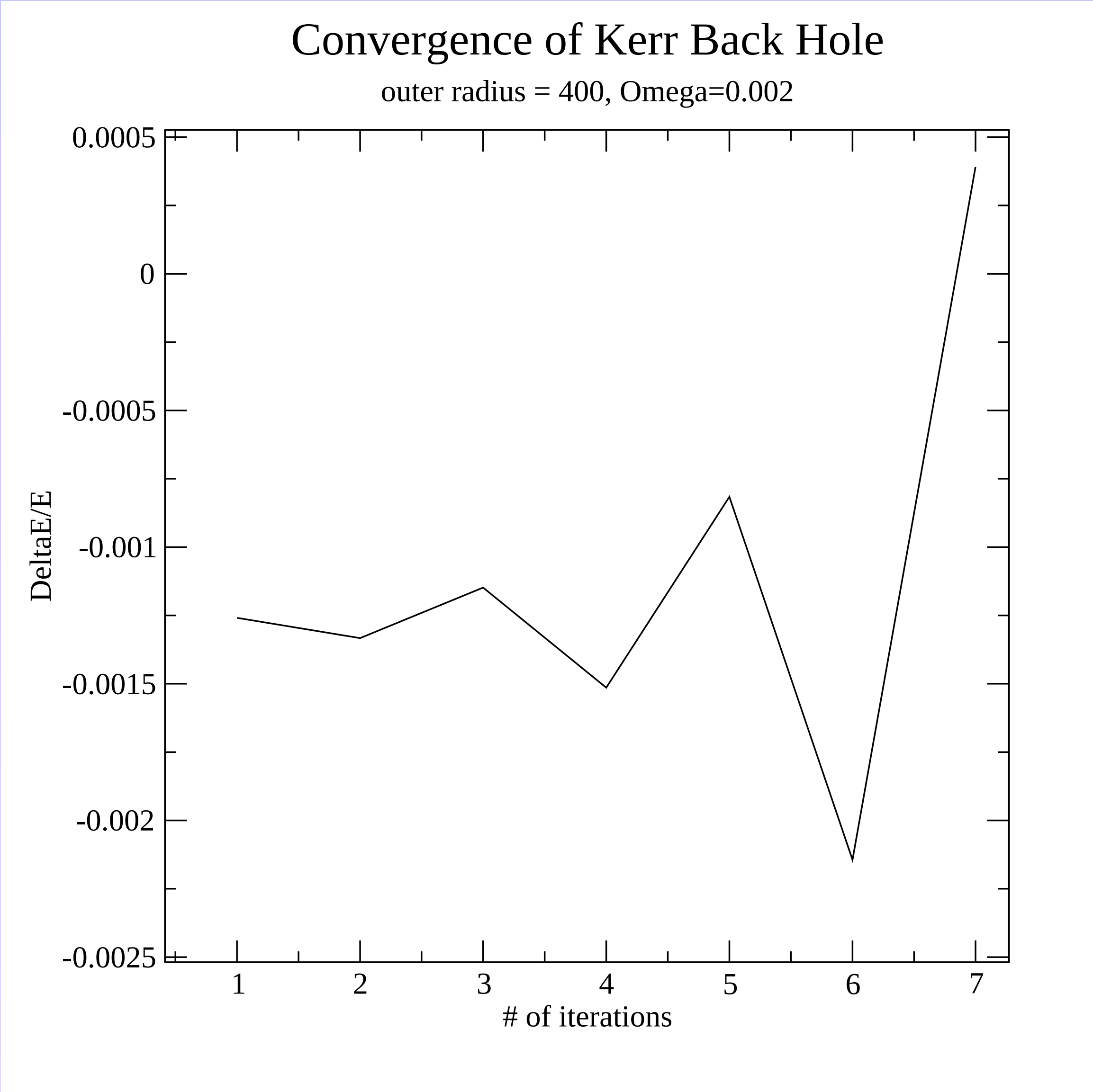
Schwarzschild

- The determinant of the conformal metric is a normalization condition imposed on the equations.
- Numerically, the determinant should converge to one as the number of radial data points are increased.





Calculating Kerr



Parameters

- R = 9, L = 8
- outer radius of domain = 400
- $\delta E = E_{Kerr} - E_{ADM}$

Future Work

- Implement proper boundary conditions
- Deal with ellipticity issues
- Long Term - try to get useable non-flat BBHID

Questions, Thanks and References

Questions?

References

[1] J. L. Friedman, K. Uryū, and M. Shibata. Deriving formulations for numerical computation of binary neutron stars in quasicircular orbits. *Phys. Rev. D*, 70:044044/1–18, Aug. 2004. 4

[2] K. Uryū, F. Limousin, J. L. Friedman, E. Gourgoulhon, and M. Shibata. Binary neutron stars: Equilibrium models beyond spatial conformal flatness. *Phys. Rev. Lett.*, 97:171101/1–4, Oct. 2006. 4