Some Issues Regarding Cosmic Acceleration

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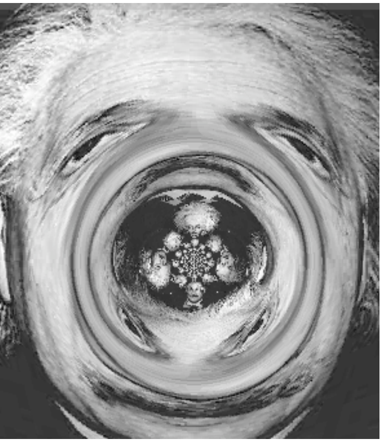
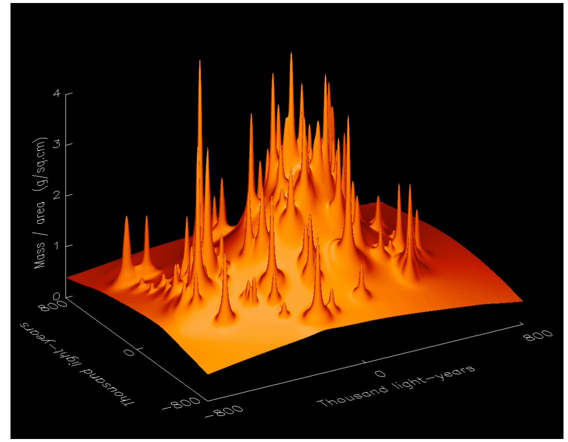
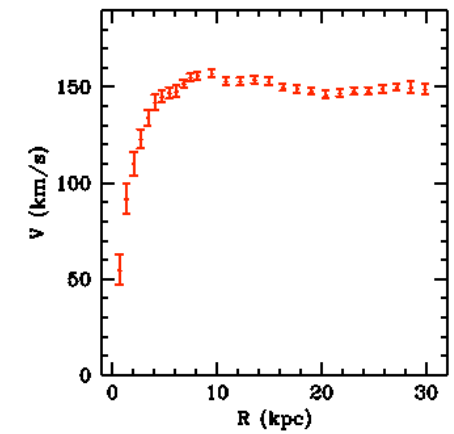
Outline

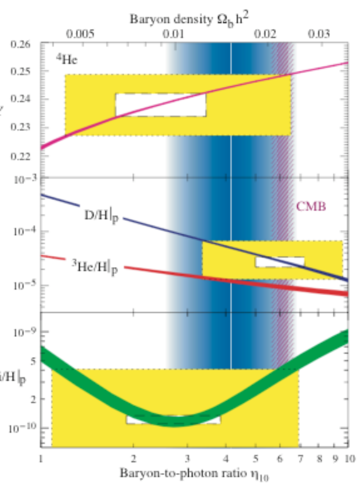
- Introduction.
- The Universe Observed, Circa 2007
- The structure of the dark sector
    - dark matter
    - dark energy/modified gravity
- Couplings in the dark sector
    - The adiabatic regime
    - The adiabatic instability
- Constraints and an Example
- Summary and Conclusions

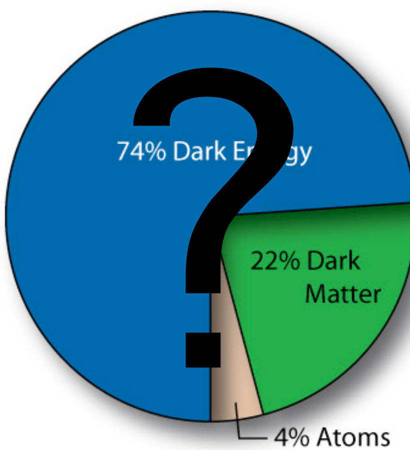
Establishing the New Cosmology

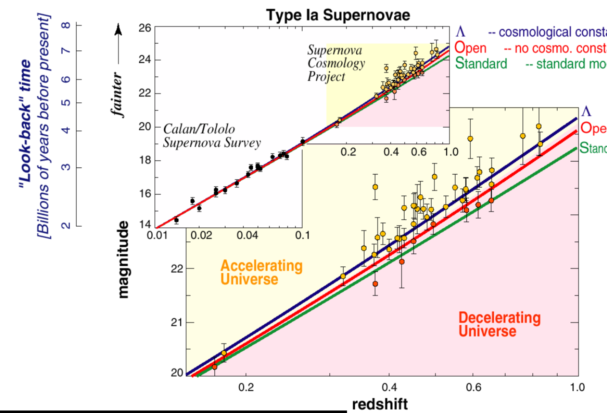


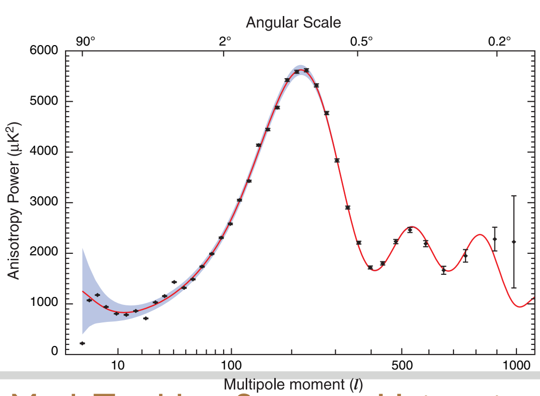


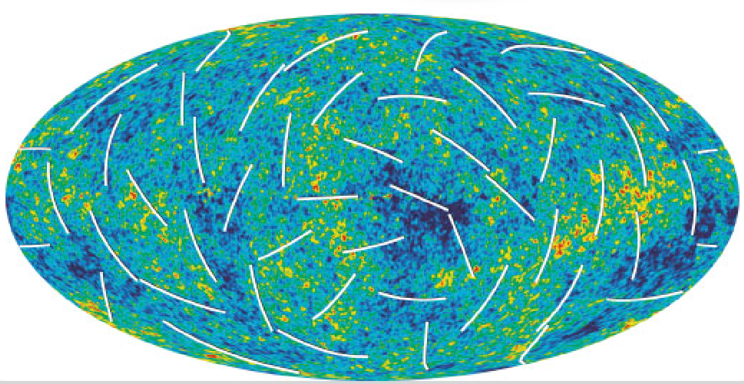


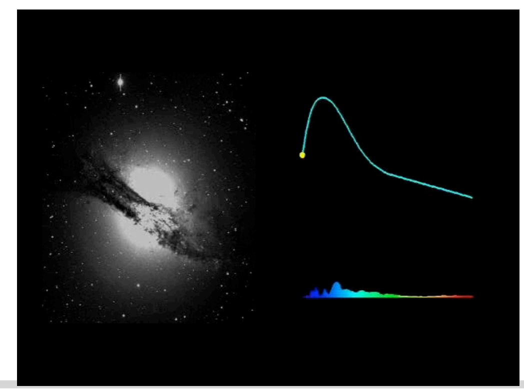












Dark Energy - Theory

Evolution of the universe governed by Einstein eqns

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 \propto \rho$$ The Friedmann equation

$$\frac{\ddot{a}}{a} \propto -(\rho + 3p)$$ The "acceleration" equation

Parameterize different types of matter by equations of state: $p_i = w_i \rho_i$

When evolution dominated by type i, obtain

$$a(t) \propto t^{2/3(1+w_i)} \qquad \rho(a) \propto a^{-3(1+w_i)}$$ $(w_i \neq -1)$

Cosmic Acceleration

$$\frac{\ddot{a}}{a} \propto -(\rho + 3p)$$

So, accelerating expansion means

p<-ρ/3 or $\boxed{\text{w<-1/3}}$

Three Broad Possibilities $a(t) \propto t^{2/3(1+w_i)}$ $\rho(a) \propto a^{-3(1+w_i)}$

	-1<w<-1/3	w=-1	w<-1
Evolution of Energy Density	Dilutes slower than any matter	Stays absolutely constant (Λ)	Increases with the expansion!!
Evolution of Scale Factor	Power-law quintessence	Exponential expansion	Infinite value in a finite time!!

Logical Possibilities for Acceleration

Modifications of Gravity

Inverse Curvature Gravity
DGP Braneworlds
Cardassian Models
...

[Carroll, Duvvuri, Trodden, Turner; Dvali, Gabadadze, Porrati; Freese, Lewis; De Felice, Easson; Ahmed, Dodelson, Sorkin; Flanagan; Moffat; ...]

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Other

Cosmological Constant
Extra Dimensions
Backreaction
Environmental Selection
...

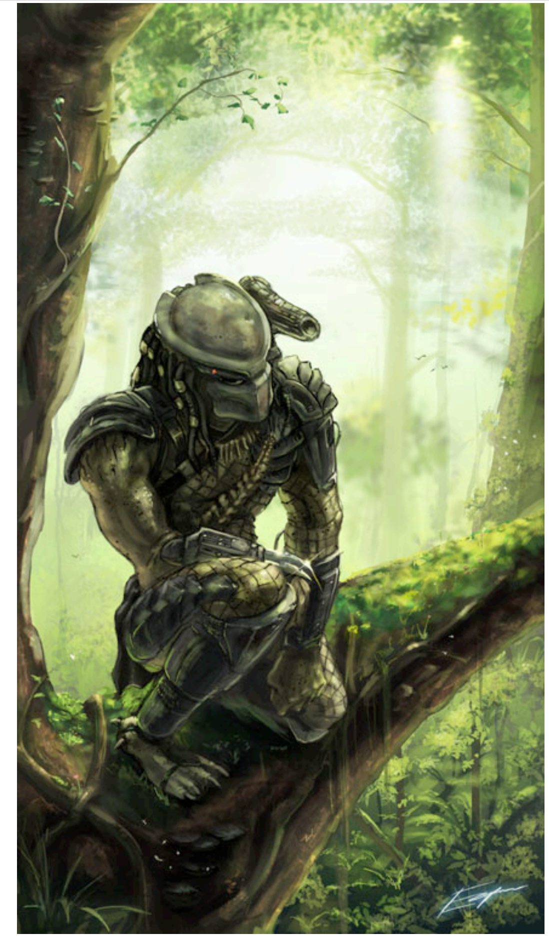
[Kolb, Matarrese, Notari, Riotto; Brandenberger; Abramo, Woodard; Weinberg; Vilenkin; Linde; Bousso, Polchinski; ...]

New Mass/Energy Sources

Quintessence
K-essence
Oscillating DE
...

[Ratra, Peebles; Wetterich; Caldwell, Dave, Steinhardt; Freiman, Hill, Stebbins, Waga; Armendariz-Picon, Mukhanov; Khoury; Bean; ...
....
Basically every cosmologist you can think of ... and most particle theorists as well.]

A Cosmological Constant or Not?

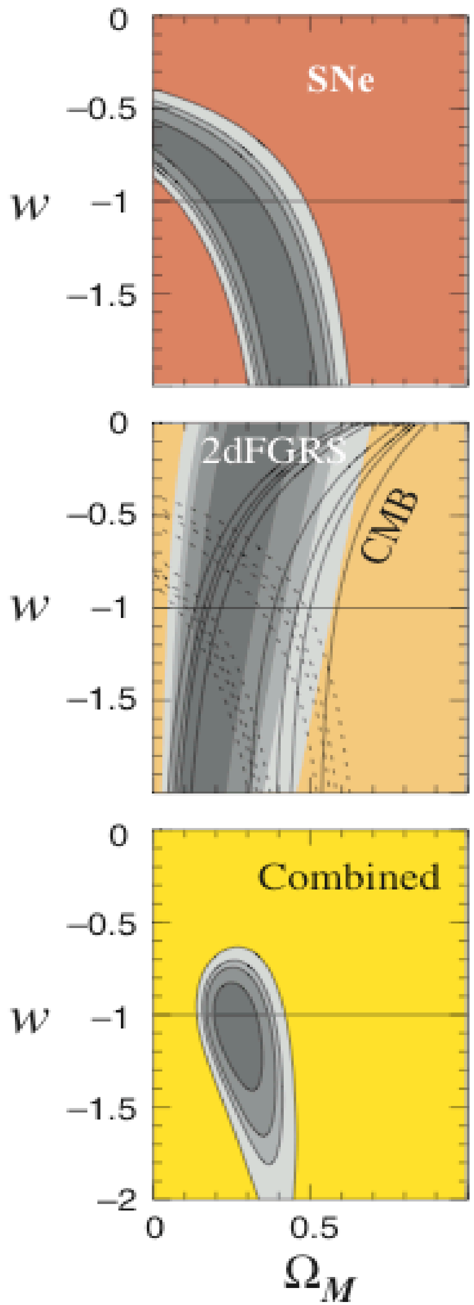


“If it bleeds, we can kill it”

Arnold Schwarzenegger as Dutch Schaeffer, *Predator* (1987)

What the Governator is trying to tell us is that our best chance of testing the origin of acceleration is if it is ***not*** a cosmological constant

Data on w



Supernova Cosmology Project
Knop *et al.* (2003)

Assuming constant w

With limits from;
2dFGRS (Hawkins et al. 2002)
and CMB (Bennet et al. 2003,
Spergel et al. 2003)

$$w = -1.05 \; ^{+0.15}_{-0.20} \text{ (statistical)} \; \pm 0.09 \text{ (systematic)}$$

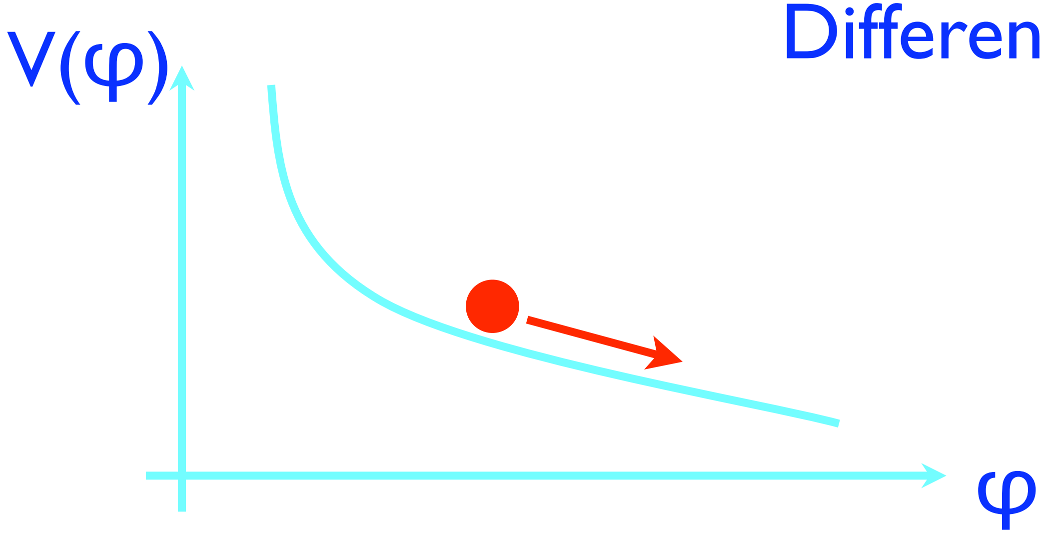
We'll come back later to discuss what is really being measured here

Quintessence - Dark Energy

Maybe there's some principle that sets vacuum energy to zero. Then dark energy might be like low-scale inflation today.

Use scalar fields to source Einstein's equation.

Difference: no minimum or reheating



$$L = \frac{1}{2}(\partial_\mu \phi)\partial^\mu \phi - V(\phi)$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi)$$

Homogeneity gives

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Small slope $\rho_\phi \approx V(\phi) \approx constant$

$$w = -\left[\frac{2V(\phi) - \dot{\phi}^2}{2V(\phi) + \dot{\phi}^2}\right]$$

Are we Being Fooled by Gravity?

(Carroll, De Felice & M.T., *Phys.Rev.* **D71:** 023525 (2005) [astro-ph/0408081])

We don't *really* measure w - we infer it from the Hubble plot via

$$w_{eff} = -\frac{1}{1-\Omega_m}\left(1+\frac{2}{3}\frac{\dot{H}}{H^2}\right)$$

Maybe, if gravity is modified, can infer value not directly related to energy sources (or perhaps without them!)

One example - Brans-Dicke theories

$$\boxed{S_{BD} = \int d^4x\sqrt{-g}\left[\phi R - \frac{\omega}{\phi}(\partial_\mu\phi)\partial^\mu\phi - 2V(\phi)\right] + \int d^4x\sqrt{-g}L_m(\psi_i, g)}$$

$\omega > 40000$ (Signal timing measurements from Cassini)

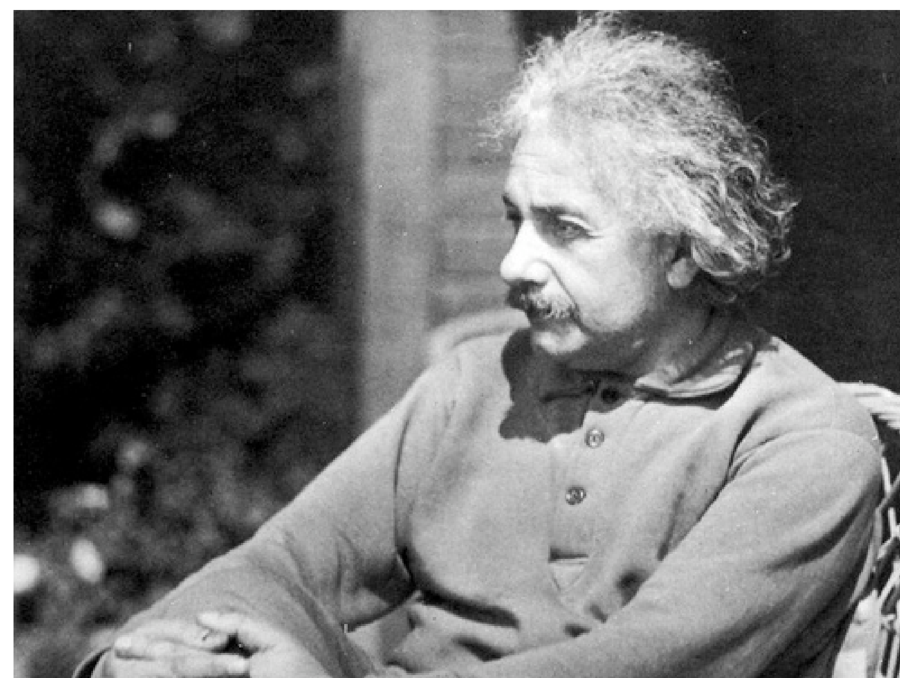
We showed that (with difficulty) can measure $w < -1$, even though no energy conditions are violated.

In Fact, Maybe it's All Gravity!

Annales de l'Observatoire Impérial de Paris. Publiées par U. J. Leverrier, Directeur de l'Observatoire, tom. v. 4to, Paris, 1859.

This volume contains the theory and tables of *Mercury* by M. Leverrier; the discrepancy as regards the secular motion of the perihelion which is found to exist between theory and observation, led, as is well known, to the suggestion by M. Leverrier of the existence of a planet or group of small planets interior to *Mercury*. The volume contains also a memoir by M. Foucault, on the " Construction of Telescopes with Silvered

"[General Relativity] explains ... quantitatively ... the secular rotation of the orbit of Mercury, discovered by Le Verrier, ... without the need of any special hypothesis.", **SPAW, Nov 18, 1915**



How Might We Modify Gravity?

Write an Lagrangian - a scalar involving the object $g_{\mu\nu}$ and its derivatives. What might this look like?

What are the propagating degrees of freedom? A first step is to identify the degrees of freedom in $g_{\mu\nu}$

Answer: turns out there are scalars and vectors as well as $h_{\mu\nu}$

How come we don't see all these in GR? - Depends on the action!

The equations of motion arising from the Einstein-Hilbert action yield constraints, which make everything except $h_{\mu\nu}$ non-dynamical!

Almost any other action will free up some of the other degrees of freedom. These can yield new problems.

Issues with new d.o.f.

A couple of different problems can arise with these new degrees of freedom.

First: Geodesics within the solar system can be appreciably altered

Best tests are from timing delays of signals from distant spacecraft.
Particularly the Cassini mission.

Second: They can lead to instabilities because they are ghost-like (have the wrong sign kinetic terms (as in more general actions - Carroll, De Felice, Duvvuri, Easson, M.T. & Turner, Phys.Rev. D71: 063513 (2005) [arXiv:astro-ph/0410031]).

These would lead, among other things, to the decay of the vacuum on a microscopic timescale
[c.f. *phantom* models!]

(Carroll, Hoffman & M.T., *Phys.Rev.* **D68:** 023509 (2003) [astro-ph/0301273])

(De Felice, Hindmarsh and M.T., JCAP 0608:005, (2006) [astro-ph/0604154])

Modifying Gravity - an Example

(Carroll, Duvvuri, M.T. & Turner, *Phys.Rev.* **D70:** 043528 (2004) [astro-ph/0306438])

Consider modifying the Einstein-Hilbert action

$$S = \frac{M_P^2}{2}\int d^4x\,\sqrt{-g}\,f(R) + \int d^4x\,\sqrt{-g}\,L_m$$

e.g. $$f(R) = R - \frac{\mu^{2(n+1)}}{R^n}$$

This is the important bit!

Field equation (n=1):

$$\boxed{\left(1+\frac{\mu^4}{R^2}\right)R_{\mu\nu} - \frac{1}{2}\left(1-\frac{\mu^4}{R^2}\right)Rg_{\mu\nu} + \mu^4\left[g_{\mu\nu}\nabla_\alpha\nabla^\alpha - \nabla_{(\mu}\nabla_{\nu)}\right]R^{-2} = \frac{T^M_{\mu\nu}}{M_P^2}}$$

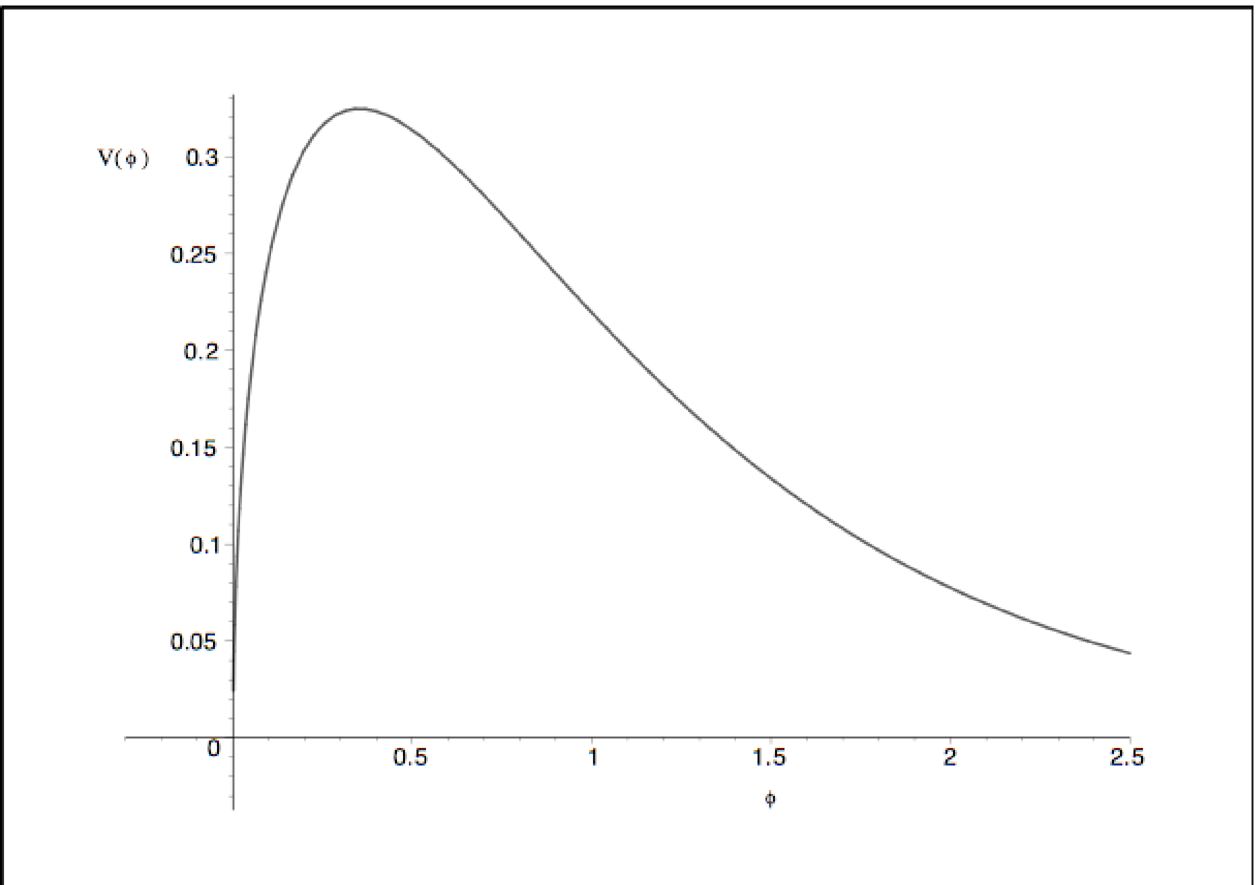
With, for cosmology

$$T^M_{\mu\nu} = (\rho_M + P_M)U_\mu U_\nu + P_M g_{\mu\nu}$$

Einstein-Frame Dynamics

$$\tilde{H}^2 = \frac{1}{3M_P^2}\left[\rho_\phi + \tilde{\rho}_M\right] \qquad \rho_\phi = \frac{1}{2}\phi'^2 + V(\phi)$$

$$\phi'' + 3\tilde{H}\phi' + \frac{dV}{d\phi}(\phi) - \frac{(1-3w)}{\sqrt{6}}\tilde{\rho}_M = 0 \qquad \tilde{\rho}_M = \frac{C}{\tilde{a}^{3(1+w)}}\exp\left[-\frac{(1-3w)}{\sqrt{6}}\frac{\phi}{M_P}\right]$$

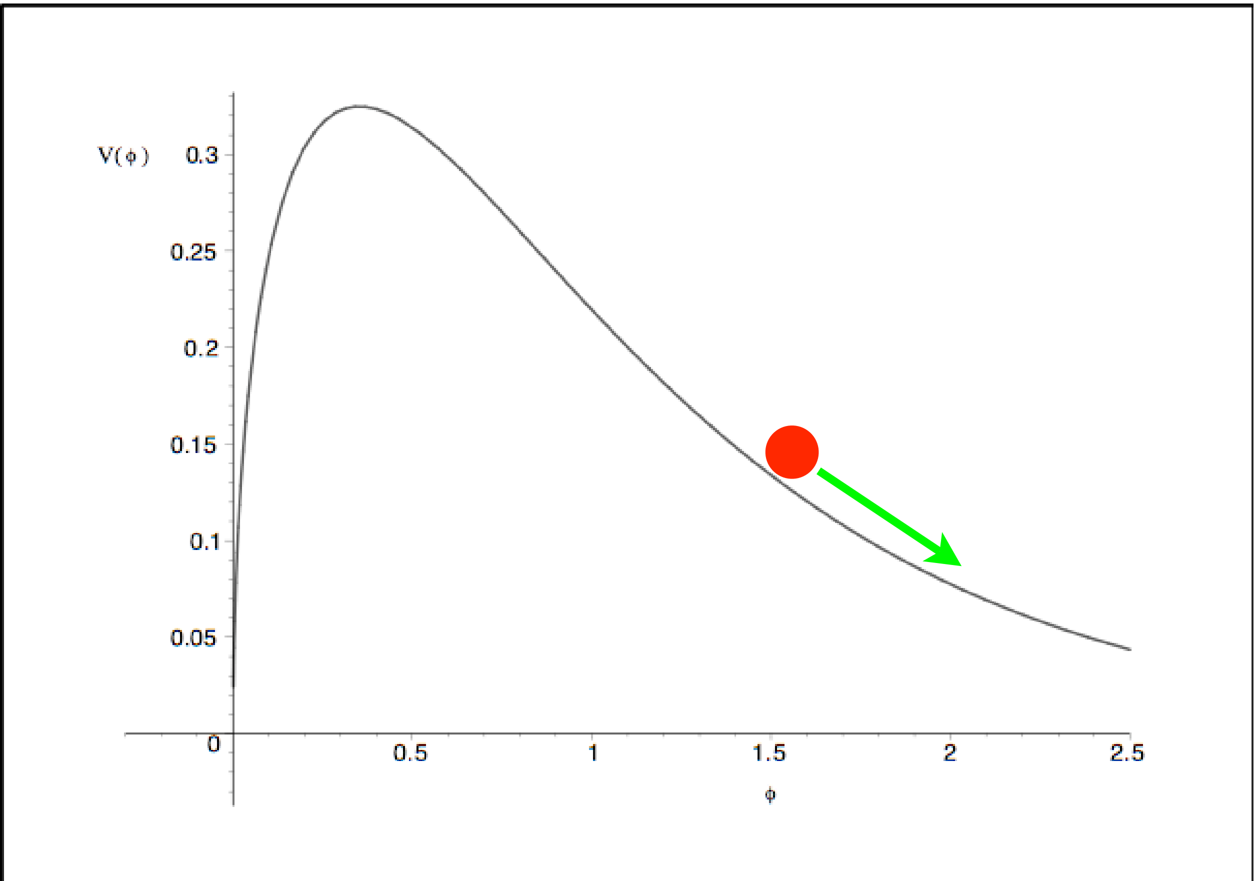


$$V(\phi) = \mu^2 M_P^2 e^{-2\beta\phi}\sqrt{e^{\beta\phi} - 1}$$

A Surprise Possibility

This frees up precisely one new degree of freedom φ

Then, φ can roll down potential and asymptotic solution is easy to find… Power-law acceleration! $a(t) \propto t^2$



Like having instantaneous equation of state parameter

$$w_{eff} = -\frac{2}{3}$$

Facing the (Solar System) Data

Easy to see model has problems agreeing with GR on scales smaller than cosmology. Can map theory to

$$S_{BD} = \int d^4x\sqrt{-g}\left[\phi R - \frac{\omega}{\phi}(\partial_\mu\phi)\partial^\mu\phi - 2V(\phi)\right] + \int d^4x\sqrt{-g}L_m(\psi_i, g)$$

i.e., a Brans-Dicke theory, with a potential that we may ignore, with ω=0

But, solar system measurements constrain ω>40000

More complicated versions survive, although constraints are strict.

Facing the (Cosmology) Data

Recall that data yields -1.45< w_{eff} <-0.74 but I was using n=1 for illustrative purposes. In general have

$$S = \frac{M_P^2}{2}\int d^4x\,\sqrt{-g}\left(R - \frac{\mu^{2(n+1)}}{R^n}\right) + \int d^4x\,\sqrt{-g}L_m$$

Analysis is very similar for n>1, with similar potential in Einstein frame. Yields, in matter frame

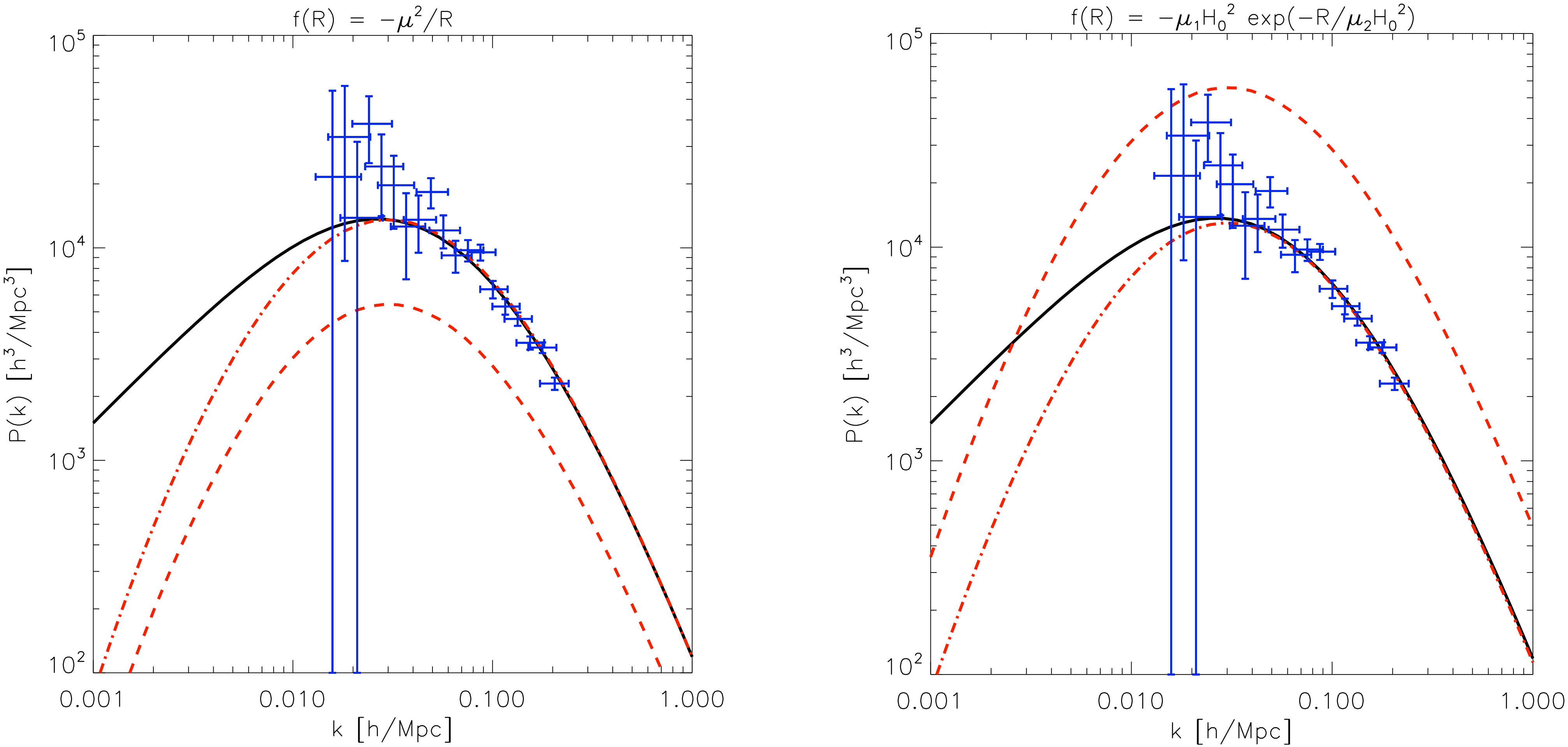
$a(t) \propto t^q$ with $q = \frac{(2n+1)(n+1)}{n+2}$

Again, this is like having $w_{eff} = -1 + \frac{2(n+2)}{3(2n+1)(n+1)}$

Easily fits data for many n (approaches dS for n$\to\infty$)

Structure Formation in f(R) Gravity

(Bean, Bernat, Pogosian, Silvestri & M.T., *Phys.Rev.* **D75:** 064020 (2007) [arXiv:astro-ph/0611321])



Matter power spectrum: ΛCDM (full black); two f(R) models for same normalization (red dashed) and normalized to give small scale agreement (red dot-dashed). Data points are SDSS data

f(R) doesn't simultaneously give small scale agreement with galaxy matter power spectrum and large scale agreement with the CMB.

Coupled Dark Models

If we evade instabilities, one frequently ends up with a model with some new component coupled nontrivially to DM and/or baryons.

There exist many examples:

O.E. Bjaelde, A.W. Brookfield, C. van de Bruck, S. Hannestad, D.F. Mota, L. Schrempp and D.Tocchini-Valentini, *Neutrino Dark Energy – Revisiting the Stability Issue*, arXiv:0705.2018 [astro-ph].

J. Khoury, A.Weltman, *Chameleon fields: Awaiting surprises for tests of gravity in space, Phys. Rev. Lett.* **93**, 171190 (2004).

J. Khoury and A.Weltman, *Chameleon cosmology, Phys. Rev.* **D69**, 044026 (2004).

D.F. Mota and D.J. Shaw, *Strongly coupled chameleon fields: New horizons in scalar field theory, Phys. Rev. Lett.* **97**, 151102 (2006)

R. Fardon, A.E. Nelson and N.Weiner, *Dark energy from mass varying neutrinos, JCAP* **0410**, 005 (2004)

R. Fardon, A.E. Nelson and N.Weiner, *Supersymmetric theories of neutrino dark energy, JHEP* **0603**, 042 (2006)

D.B. Kaplan, A.E. Nelson and N.Weiner, *Neutrino oscillations as a probe of dark energy, Phys. Rev. Lett.* **93**, 091801 (2004)

S.M. Carroll, V. Duvvuri, M.Trodden and M.S.Turner, *Is cosmic speed-up due to new gravitational physics?, Phys. Rev.* **D70**, 043528 (2004)

S.M. Carroll, I. Sawicki, A. Silvestri and M.Trodden, *Modified-Source Gravity and Cosmological Structure Formation, New J. Phys.* **8**, 323 (2006)

G.R. Farrar and P.J.E. Peebles, *Interacting dark matter and dark energy, Astrophys. J.* **604**, 1 (2004)

(not comprehensive!)

Modeling Dark Couplings

Consider writing a general action as

$$S = S[g_{ab}, \phi, \Psi_j] = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_p^2 R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right] + \sum_j S_j[e^{2\alpha_j(\phi)} g_{ab}, \Psi_j]$$

We'll focus just on DM/DE couplings here $\alpha_j(\phi) = \alpha(\phi)$

Model matter as a perfect fluid. EOMs become

$$M_{pl}^2 G_{ab} = \nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}(\nabla\phi)^2 - V(\phi)g_{ab} + e^{4\alpha(\phi)}\left[(\bar{\rho}+\bar{p})u_a u_b + \bar{p}g_{ab}\right]$$

$$\nabla_a\nabla^a\phi - V'(\phi) = \alpha'(\phi)e^{4\alpha(\phi)}(\bar{\rho} - 3\bar{p})$$

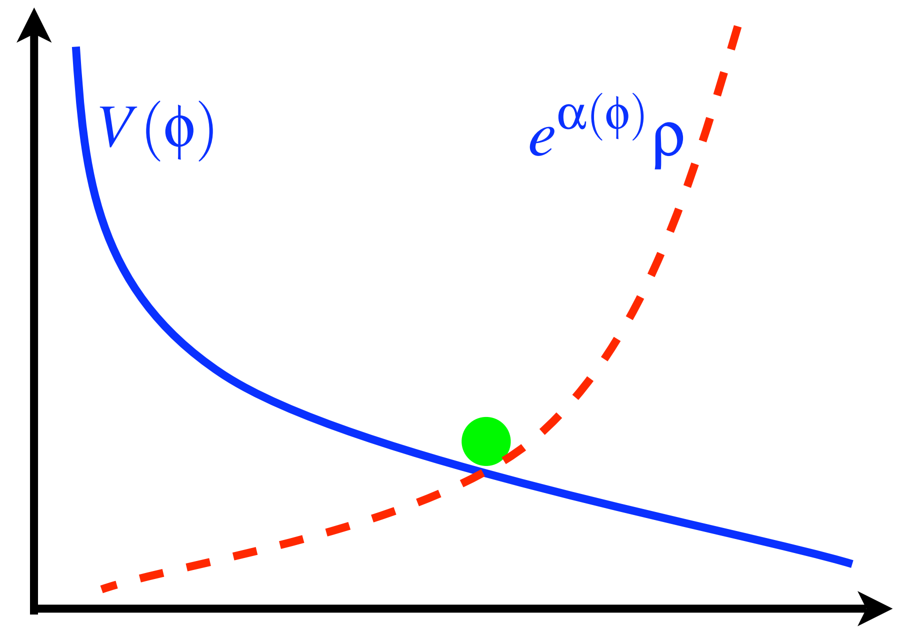
The Adiabatic Regime

Interested in a particular regime:

DE field adiabatically tracks minimum of effective potential

Define background solution $\phi_m(\rho)$ by

$$V'_{eff}(\phi) = V'(\phi) + \alpha'(\phi)e^{\alpha(\phi)}\rho = 0$$



So effective equations of motion are now

$$M_{pl}^2 G_{ab} = [(\rho_{eff} + p_{eff})u_a u_b + p_{eff} g_{ab}]$$

$$\rho_{eff}(\rho) = e^{\alpha[\phi_m(\rho)]}\rho + V[\phi_m(\rho)]$$

$$p_{eff}(\rho) = -V[\phi_m(\rho)]$$

Observed Value of w

$$w_{obs}(a) = -1 - \frac{1}{3}\frac{d\ln\rho_{DE}}{d\ln a}$$

[This is an important point in general when it comes to interpreting cosmic acceleration]

$$w_{obs} = \frac{-1}{1 - \frac{d\ln V}{d\alpha}\left(1 - e^{\alpha_0 - \alpha}\right)}$$

Note: w<-1 today, *for all models*!

Also, can expand about a=1

$$\frac{1}{w_{obs}} \simeq -1 + \ln(V/V_0)$$

So w<-1 in the recent past, *for all models*!

Validity of the Adiabatic Regime

Local Adiabatic Condition: consider a perturbation with timescale or lengthscale L and density ρ

Rough necessary condition for LAC is

$$L \gg m_{eff}^{-1}(\rho) \equiv \left(\frac{\partial^2 V_{eff}}{\partial \phi^2}(\phi, \rho)\bigg|_{\phi=\phi_m(\rho)} \right)^{-1/2}$$

More precisely, LAC holds if

$$\frac{d \ln V}{d \ln \rho} \left(\frac{1}{m_{eff}^2 L^2} \right) \ll 1$$

If holds everywhere, adiabatic approx. holds. If not, can fail nonlocally

The Adiabatic Instability

(Bean, Flanagan and M.T., [arxiv:0709.1124], [arxiv:0709.1128])

There exist several ways to look at the instability that may occur in the adiabatic regime. We'll just briefly touch on them.

The Hydrodynamic Viewpoint

Recall that $$\rho_{eff} = V + e^{\alpha}\rho = V - \frac{dV/d\phi}{d\alpha/d\phi} = V - \frac{dV}{d\alpha}$$

The adiabatic sound speed is then given by

$$\frac{1}{c_a^2} = \frac{d\rho_{eff}}{dp_{eff}} = \frac{d\rho_{eff}/d\alpha}{dp_{eff}/d\alpha} = \frac{\frac{d}{d\alpha}\left[V - \frac{dV}{d\alpha}\right]}{\frac{d}{d\alpha}\left[-V\right]} = -1 + \frac{\frac{d^2V}{d\alpha^2}}{\frac{dV}{d\alpha}}$$

In adiabatic regime $c_s^2(k,a) \equiv \frac{\delta P(k,a)}{\delta\rho(k,a)} \rightarrow c_a^2$

Instability if $c_s^2 < 0$ occurs if $c_a^2 < 0$

Can then show are in adiabatic regime and adiabatically unstable in two regions of parameter space

$$\frac{dV}{d\alpha} < \frac{d^2V}{d\alpha^2} < 0 \qquad \text{or} \qquad \frac{dV}{d\alpha} < 0 < \frac{d^2V}{d\alpha^2}$$

Instability operates in a range of scales. Must be long enough and must grow on a time-scale less than the Hubble one

$$m_{eff}^{-1} \ll L \ll \frac{\sqrt{|c_s^2|}}{H}$$

Generally, for a fluid, instability time-scale must be shorter than the gravitational dynamical time. Can put all this together to show

$$\frac{L_{max}}{L_{min}} \leq |\alpha'[\phi_m(\rho)]|$$ Nonempty only if $|\alpha'| \gg 1$

Jeans Viewpoint

In Einstein frame instability is nothing to do with gravity - all about the scalar and the coupling.

But in Jordan frame, instability involves gravity - mediated partly by a tensor interaction and partly by a scalar interaction.

Effective Newton's constant for self interaction of dark matter

$$G_{cc} = G\left[\underbrace{1}_{\text{Tensor}} + \frac{2^2\alpha'(\phi)^2}{\underbrace{1}_{\text{Scalar}} + \frac{m_{eff}^2}{k^2}}\right]$$

At long scales $G_{cc} \approx G$ and at short ones $G_{cc} \approx G[1+2^2(\alpha')^2]$

But for $M_p|\alpha'| \gg 1$ have an intermediate range of length-scales

[Important condition, missed in lots of other treatments]

$$\frac{m_{eff}}{|\alpha'|} \ll k \ll m_{eff}$$

where
$$G_{cc} \approx G \frac{2^2(\alpha')^2}{m_{eff}^2} k^2$$

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{cc} e^{\alpha} \rho = 0$$

Hubble damping is ineffective - Jeans instability causes approximate exponential growth rather than power law growth.

e.g. 2-component DM Models

Two components. One not coupled, one coupled, exponential potential, constant coupling

$$3^2H^2 = V + e^{\alpha}\rho_{co} + \rho_c$$

Fractional amount in coupled must be small in large coupling limit

Evolution equations for fractional density perturbations in the adiabatic limit on subhorizon scales given by

$$\ddot{\delta}_j + 2H\dot{\delta}_j - 4\pi\sum_k G_{jk}\rho_k e^{\alpha_k}\delta_k = 0$$

$$\ddot{\delta}_c + 2H\dot{\delta}_c = \frac{1}{2^2}\rho_c\delta_c + \frac{1}{2^2}e^{\alpha}\rho_{co}\delta_{co}$$

$$\ddot{\delta}_{co} + 2H\dot{\delta}_{co} = \frac{1}{2M_p^2}\rho_c\delta_c + \frac{1}{2M_p^2}\left[1 + \frac{2\beta^2C^2}{1 + \frac{m_{eff}^2a^2}{k^2}}\right]e^{\alpha}\rho_{co}$$

Effective equation of state (black full line)

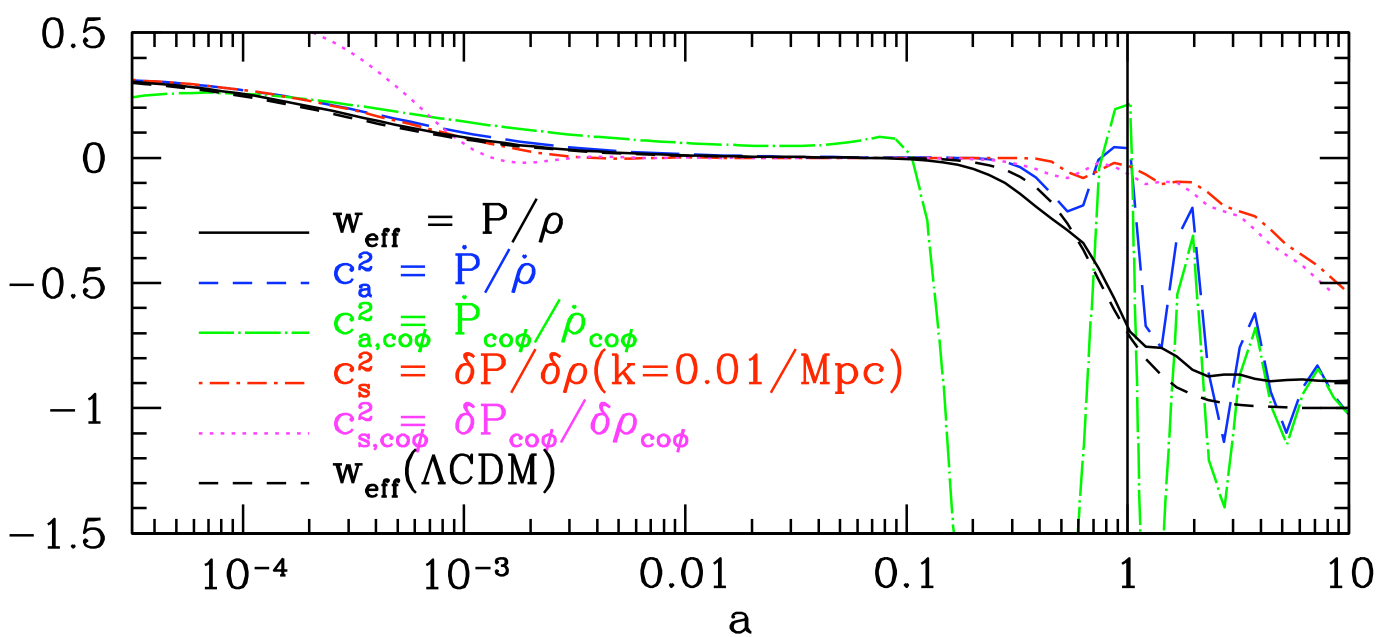
Adiabatic speed of sound, for all components (blue long dashed line)

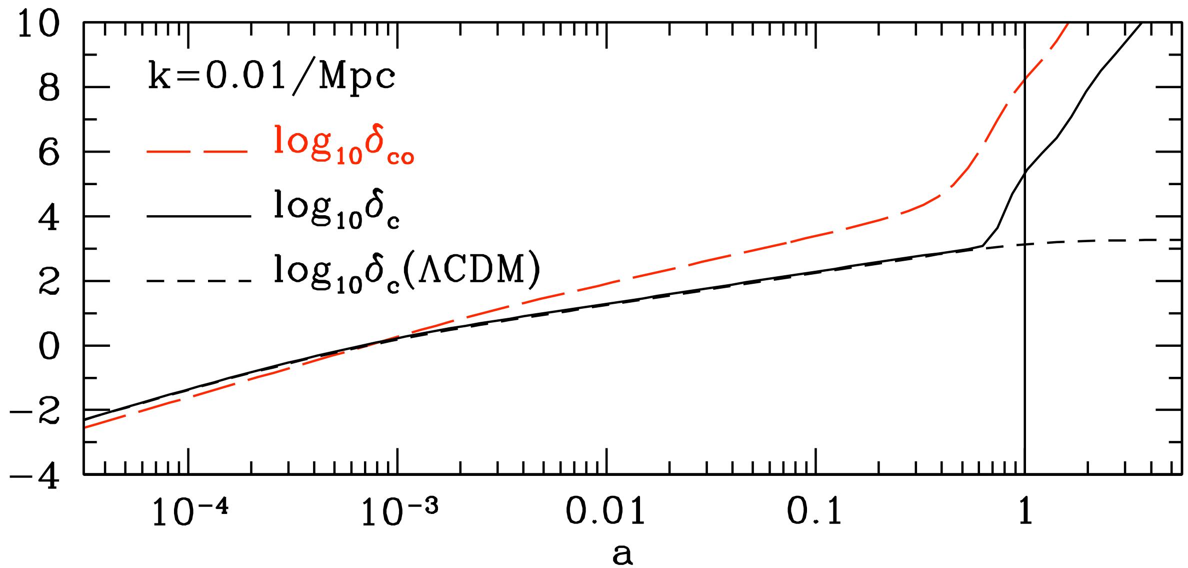
For the coupled components only (green dot long dashed line)

Effective speed of sound for all components (red dot-dashed line)

For the coupled components alone (magenta dotted line).

Effective equation of state for LCDM with $\Omega_c = 0.25$, $\Omega_b = 0.05$, and $\Omega_\Lambda = 0.05$ (black dashed line)





Fractional over-density for coupled CDM component (red long dashed line)

For uncoupled component (black full line)

For LCDM (black dashed line).

At late times adiabatic behavior triggers a dramatic increase in the rate of growth of both uncoupled and coupled components, leading to structure predictions inconsistent with observations.

2-component DM instability

These models are example of a class of theories for which

- the background cosmology is compatible with observations,
- but which are ruled out by the adiabatic instability of the perturbations.

Summary II

It may be that there are new energy components that are driving acceleration.

Or, perhaps the response of spacetime to existing matter/energy components differs from GR at long distances - modified gravity.

In general many kinds of new instabilities can exist

In many particle physics motivated proposals, there may well be nontrivial couplings, either between different energy components, or between them and new gravitational degrees of freedom.
For many such couplings, there exist environments for which the coupled system evolves adiabatically...

... and this can lead to an unacceptable instability - the adiabatic instability

Thank You!

Coupled DM/DE

For DM, need $\bar{p} = 0$

Define $\rho \equiv e^{3\alpha(\phi)}\bar{\rho}$ EOMs become

$$M_{pl}^2 G_{ab} = \nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}(\nabla\phi)^2 - V(\phi)g_{ab} + e^{\alpha(\phi)}\rho u_a u_b$$

$$\nabla_a\nabla^a\phi - V'(\phi) = \alpha'(\phi)e^{\alpha(\phi)}\rho$$

Or

$$\nabla_a\nabla^a\phi - V'_{eff}(\phi) = 0 \qquad V_{eff}(\phi) = V(\phi) + e^{\alpha(\phi)}\rho$$

$$\nabla_a(\rho u^a) = 0 \qquad u^b\nabla_b u^a = -(g^{ab} + u^a u^b)\nabla_b\alpha$$