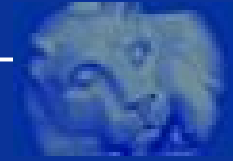




*Fundamental Properties and Phenomenological Aspects
of Loop Quantum Gravity: The Missing Link*

Martin Bojowald

The Pennsylvania State University
Institute for Gravitation and the Cosmos
University Park, PA

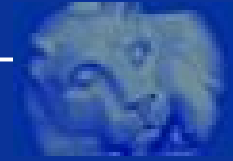


Why quantum gravity?

→ *Planck length* $\ell_P = \sqrt{G\hbar/c^3} \approx 10^{-35}\text{m}$ *tiny*, may suggest only negligible effects of order ℓ_P/L .

But: *detailed derivation required*. Example: Bohr radius $a_0 \propto \hbar^2/m_e e^2$ from dimensional arguments. Works for hydrogen, but what about heavy elements of $Z \sim 100$?

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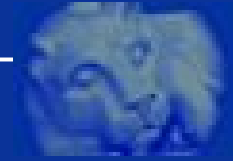
→ Big bang preceded by *singularity* in general relativity: vanishing volume in any case small compared to ℓ_P .

Crucial changes due to quantum gravity may or may not prevent singularities.

Precise form of what happens may have observational ramifications at later times.



Principles



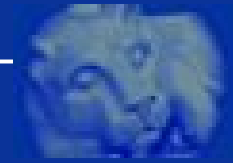
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Even before observations become available, *consistency* presents strong conditions and feedback for fundamental properties of the underlying quantum theory of gravity.

Reliably extracting these effects provides *phenomenological properties* which can eventually be used to compare with observations.



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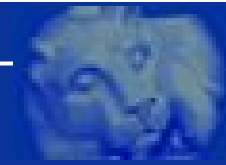
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Both fundamental and phenomenological aspects can be addressed with the concept of *effective equations*.

Not limited to correction terms in classical equations, but can be used even in strong quantum regimes such as the big bang.

Does not only provide phenomenological corrections but also a manageable analysis of consistency issues.



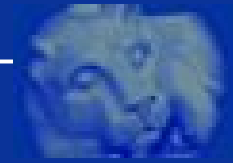
Loop quantum gravity

Status of consistency is still incomplete.

Nevertheless, has suggested *several new effects* – chiefly based on the *discreteness of spatial geometry* – which may well have observable implications.

Effective equations crucial not only to analyze the consistency further but also to bridge the gap to phenomenology by providing reliable quantum corrections.

As a canonical quantization of general relativity, it requires its own techniques for effective equations (rather than effective actions). Now available and applied to cosmological issues.

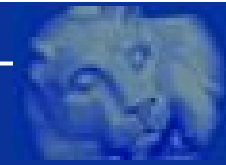


Loop variables

General relativity can be formulated as *gauge theory* in Ashtekar variables (A_a^i, E_j^b) with gauge group $SO(3)$ for rotations.

E_i^a determines *spatial geometry*: $E_i^a E_i^b = \det q q^{ab}$,

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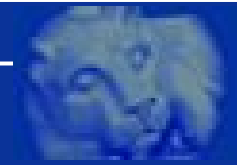
A_a^i related to *change in time of geometry* \longrightarrow *momentum*.

Define *holonomies* and *fluxes* independently of metric:

$$h_e(A) = \mathcal{P} \exp \int_e A_a^i \tau_i \dot{e}^a dt \quad , \quad F_S(E) = \int_S d^2 y n_a E_i^a \tau_i$$

for any curve e and surface S in space (τ_i : Pauli matrices)

Holonomies serve as *creation operators of geometrical excitations* measured by fluxes.



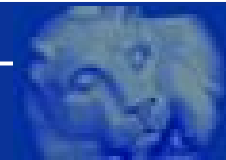
Discrete spectrum (U(1)-simplification)

Define basic state $|0\rangle$ by $\langle A_a|0\rangle = 1$: independent of holonomies.
Excited states

$$|e_1, n_1; \dots; e_i, n_i\rangle = \hat{h}_{e_1}^{n_1} \dots \hat{h}_{e_i}^{n_i} |0\rangle$$

General state labeled by *graph g and integers n_e as quantum numbers* on edges

$$\psi_{g,n}(A_a) = \prod_{e \in g} h_e(A_a)^{n_e} = \prod_{e \in g} \exp(in_e \int dt \dot{e}^a A_a)$$



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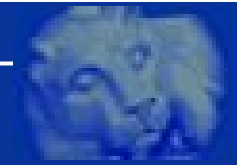
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Flux quantized as derivative operator, *measures excitation level*:

$$\begin{aligned} \hat{F}_S \psi_{g,n} &= \frac{G\hbar}{i} \int_S d^2y n_a \frac{\delta \psi_{g,n}}{\delta A_a(y)} \\ &= \ell_P^2 \sum_{e \in g} n_e \int_S d^2y \int_e dt n_a \dot{e}^a \delta(y, e(t)) h_e \frac{\partial \psi_{g,n}}{\partial h_e} = \ell_P^2 \sum_{e \in g} n_e \text{Int}(S, e) \psi_{g,n} \end{aligned}$$

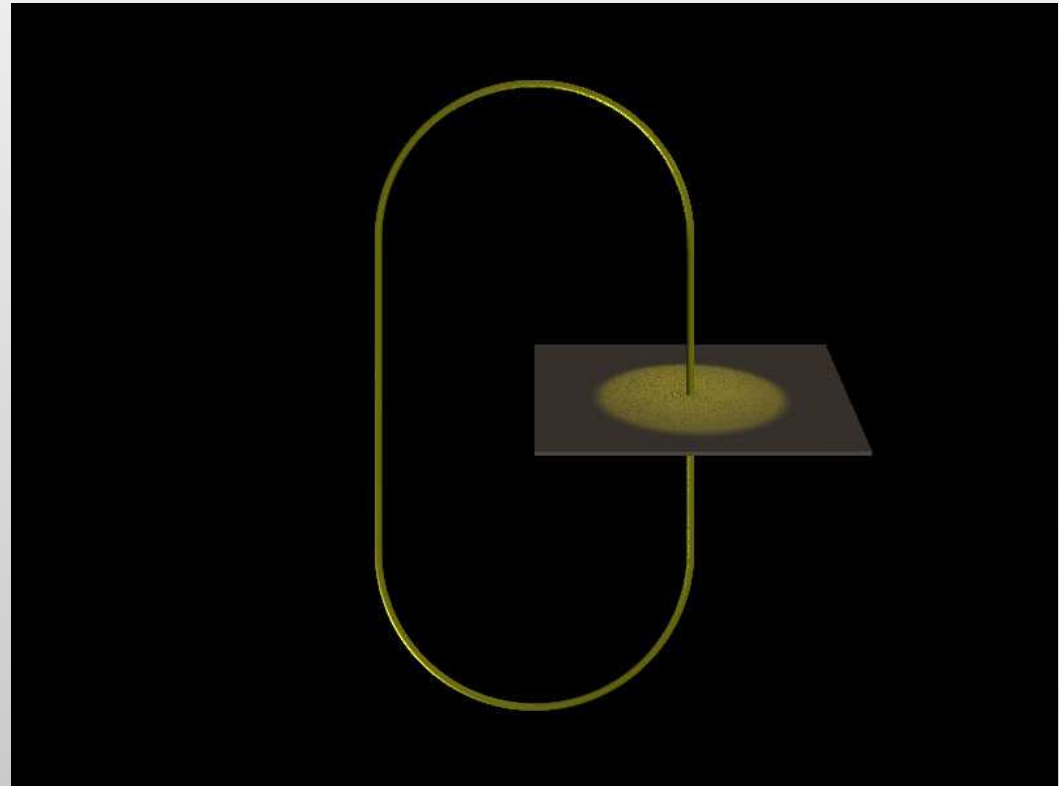
Discrete spectrum of geometry.



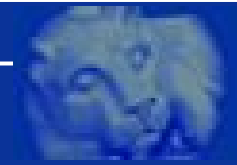
Excitations of geometry

Multiplying with holonomies for a given set of curves creates dependence on A_a^i along curves and thus geometry.

Single excitation:



Loop as visualization of state. Physical meaning through measurement of flux illustrated by intersecting surface.

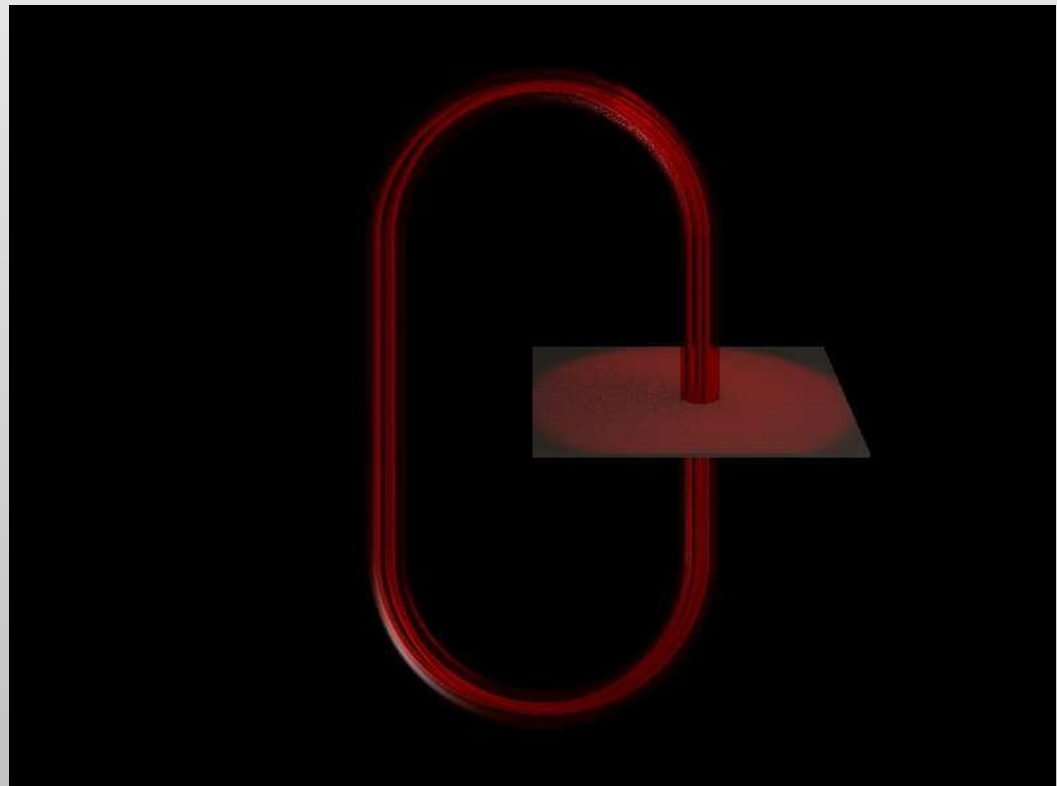


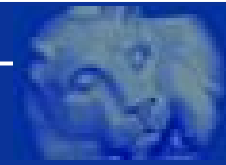
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Higher excitations in two ways:

use creation operators for the same loop





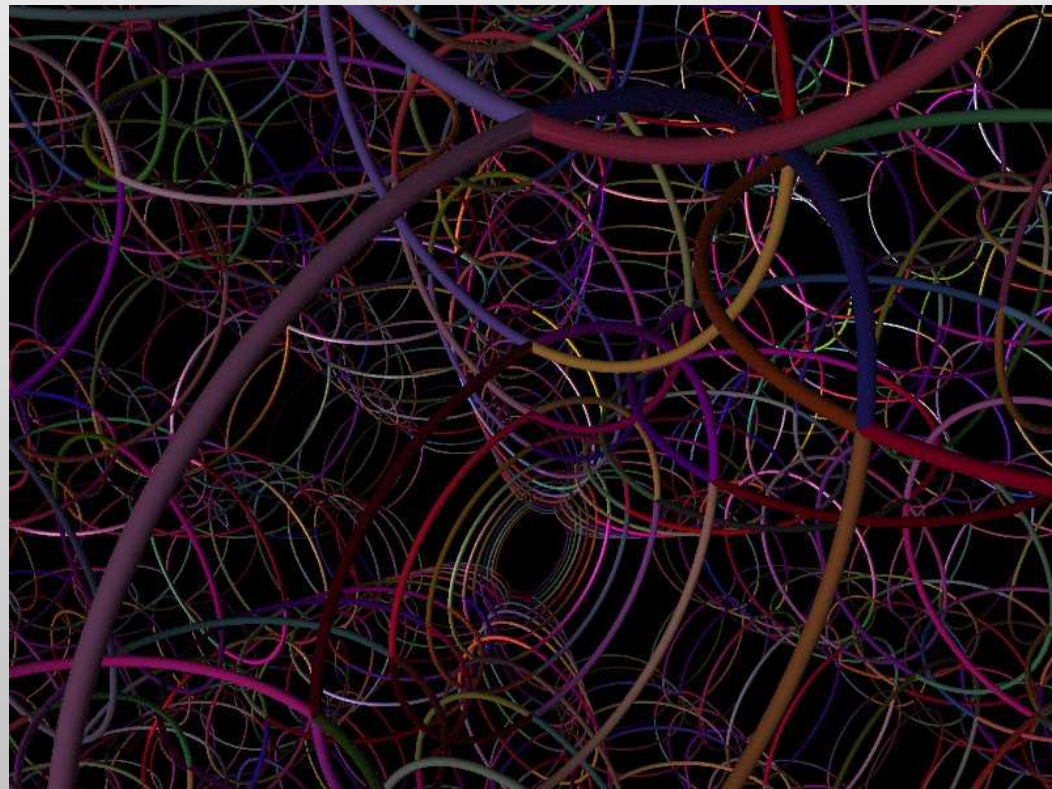
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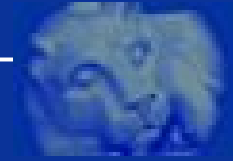
Higher excitations in two ways:

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or use different loops.



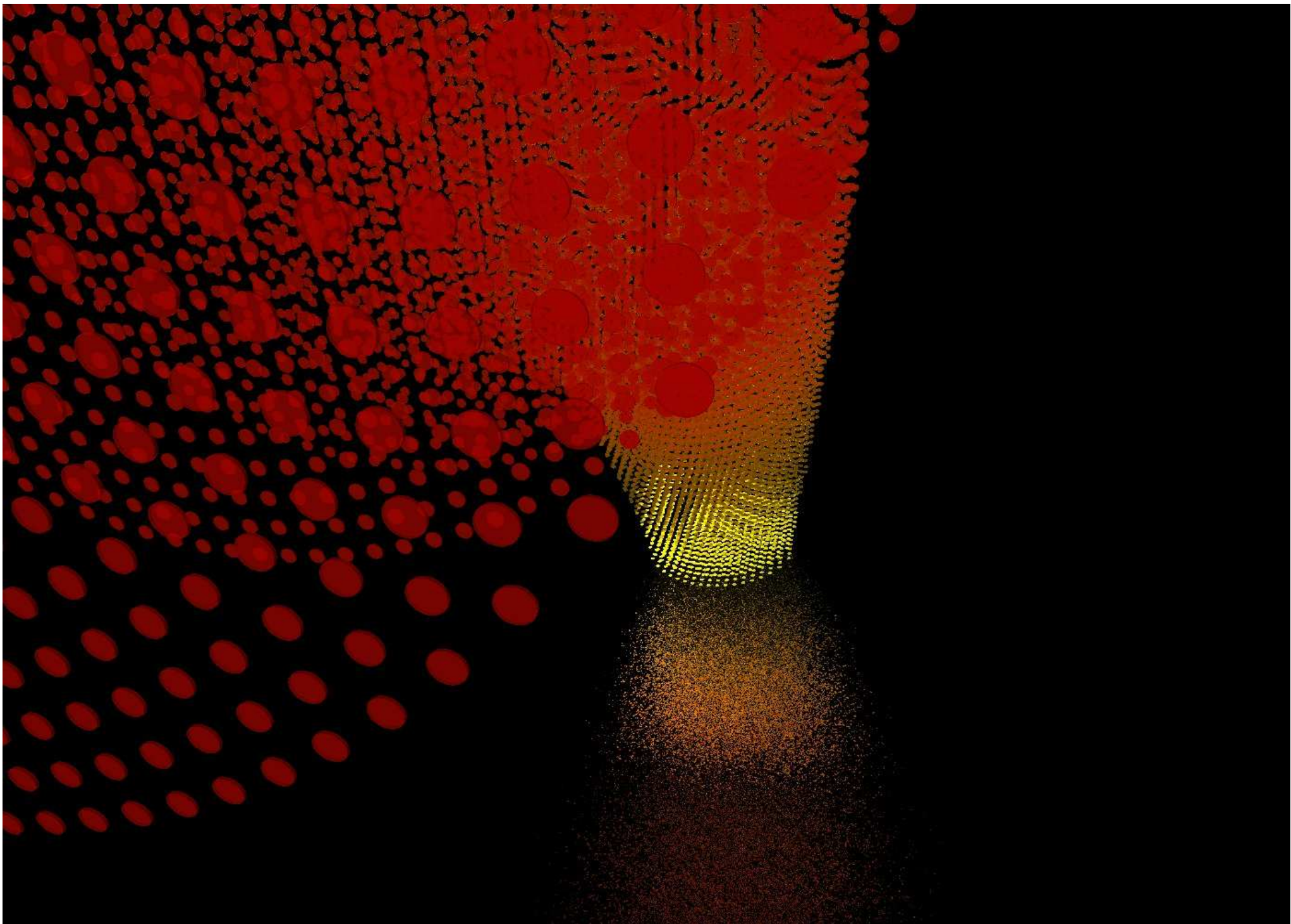
Strong excitation necessary for macroscopic geometry.

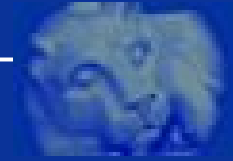


$$\hat{H} = \sum_{v,IJK} \epsilon^{IJK} \text{tr}(h_{v,I} h_{v+I,J} h_{v+J,I}^{-1} h_{v,J}^{-1} h_{v,K} [h_{v,K}^{-1}, \hat{V}])$$

as (simplified) Hamiltonian: *excitations of geometry take place dynamically*. Universe as growing crystal of discrete space: atoms of space created and excited as universe expands.

Precise balance of different forms of excitations important for details of quantum corrections. (More later.)





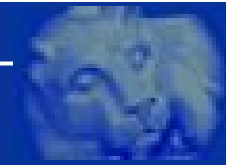
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as (simplified) Hamiltonian: *excitations of geometry take place dynamically*. Universe as growing crystal of discrete space: atoms of space created and excited as universe expands.

Precise balance of different forms of excitations important for details of quantum corrections. (More later.)

Significant at *high curvature* (big bang), but there may also be many small corrections adding up in a *large universe* (dark energy).

Dark energy as Brownian motion of quantum gravity?



Quantum corrections

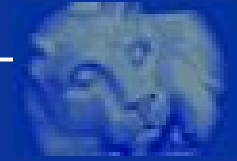
Three types of quantum corrections, in general equally important:

—→ Entire states evolve which spread and deform. Quantum fluctuations, correlations and higher moments *independent variables back-reacting* on expectation values. Quantum degrees of freedom arise, which incorporate some of the effects of higher time derivatives.

—→ Holonomies as *non-local, non-linear* functions imply corrections similar to some terms in higher curvature actions.

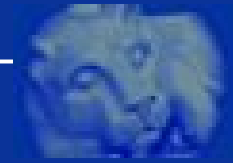
—→ *Inverse metric components* receive corrections since operators with zero in their discrete spectra have no inverse.

Can be incorporated in effective equations, taking into account fundamental aspects such as physical normalization of states.



Applications

- *Resolve big bang singularity* in isotropic models by new repulsive forces due to quantum gravity.
- Include *inhomogeneities*. Determine evolution of gauge invariant perturbations.



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—→ *Resolve big bang singularity* in isotropic models by new repulsive forces due to quantum gravity.

—→ Include *inhomogeneities*. Determine evolution of gauge invariant perturbations.

Analysis combines fundamental properties of Hamiltonian (constraint) with phenomenological properties of cosmological scenarios.

Example: gauge invariant perturbations possible only if discrete structure of space respects *general covariance*. (More later.)

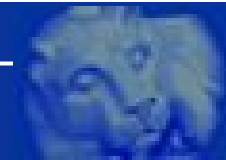


Illustration: anharmonic oscillator

Hamiltonian: $\hat{H} = \frac{1}{2m}\hat{p}^2 + V(\hat{q}) = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{q}^2 + \frac{1}{3}\lambda\hat{q}^3$

Equations of motion

$$\frac{d}{dt}\langle\hat{q}\rangle = \frac{1}{i\hbar}\langle[\hat{q}, \hat{H}]\rangle = \frac{1}{m}\langle\hat{p}\rangle$$

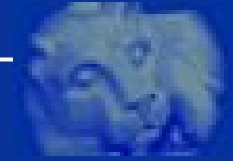
$$\frac{d}{dt}\langle\hat{p}\rangle = \frac{1}{i\hbar}\langle[\hat{p}, \hat{H}]\rangle = -V'(\langle\hat{q}\rangle) - \lambda(\Delta q)^2$$

$$\frac{d}{dt}(\Delta q)^2 = \frac{2}{m}C_{qp}$$

$$\frac{d}{dt}C_{qp} = \frac{1}{m}C_{qp} + m\omega^2(\Delta q)^2 + 6\lambda\langle\hat{q}\rangle(\Delta q)^2 + 3\lambda G^{0,3}$$

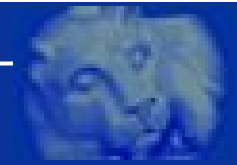
...

with covariance $C_{qp} = \frac{1}{2}\langle\hat{q}\hat{p} + \hat{p}\hat{q}\rangle - \langle\hat{q}\rangle\langle\hat{p}\rangle$ and higher moment $G^{0,3} = \langle(\hat{q} - \langle\hat{q}\rangle)^3\rangle = \langle\hat{q}^3\rangle - 3\langle\hat{q}\rangle(\Delta q)^2 - \langle\hat{q}\rangle^3$ of third order (skewness).



Effective equations

Infinite system of coupled differential equations. Manageable only if decoupled approximately. Crucial: *Solvable systems* which decouple exactly, then perturbations around them. Example: harmonic oscillator.



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Solvable system for cosmology: spatially flat cosmology with free massless scalar ϕ .

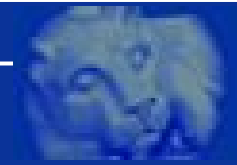
Use variables based on $\exp(ia^{2x}\dot{a})$ with $-1/2 < x < 0$: holonomies, discreteness, *refinement*. (More later.)

$$V = a^{2(1-x)} \quad , \quad J = a^{2(1-x)} \exp(ia^{2x}\dot{a})$$

as non-canonical position and momentum:

$$[\hat{V}, \hat{J}] = \hbar \hat{J} \quad , \quad [\hat{V}, \hat{J}^\dagger] = -\hbar \hat{J}^\dagger \quad , \quad [\hat{J}, \hat{J}^\dagger] = -2\hbar \hat{V} - \hbar^2$$

Hamiltonian (in ϕ) $\hat{H} = \frac{1}{2i}(\hat{J} - \hat{J}^\dagger)$ *linear*.



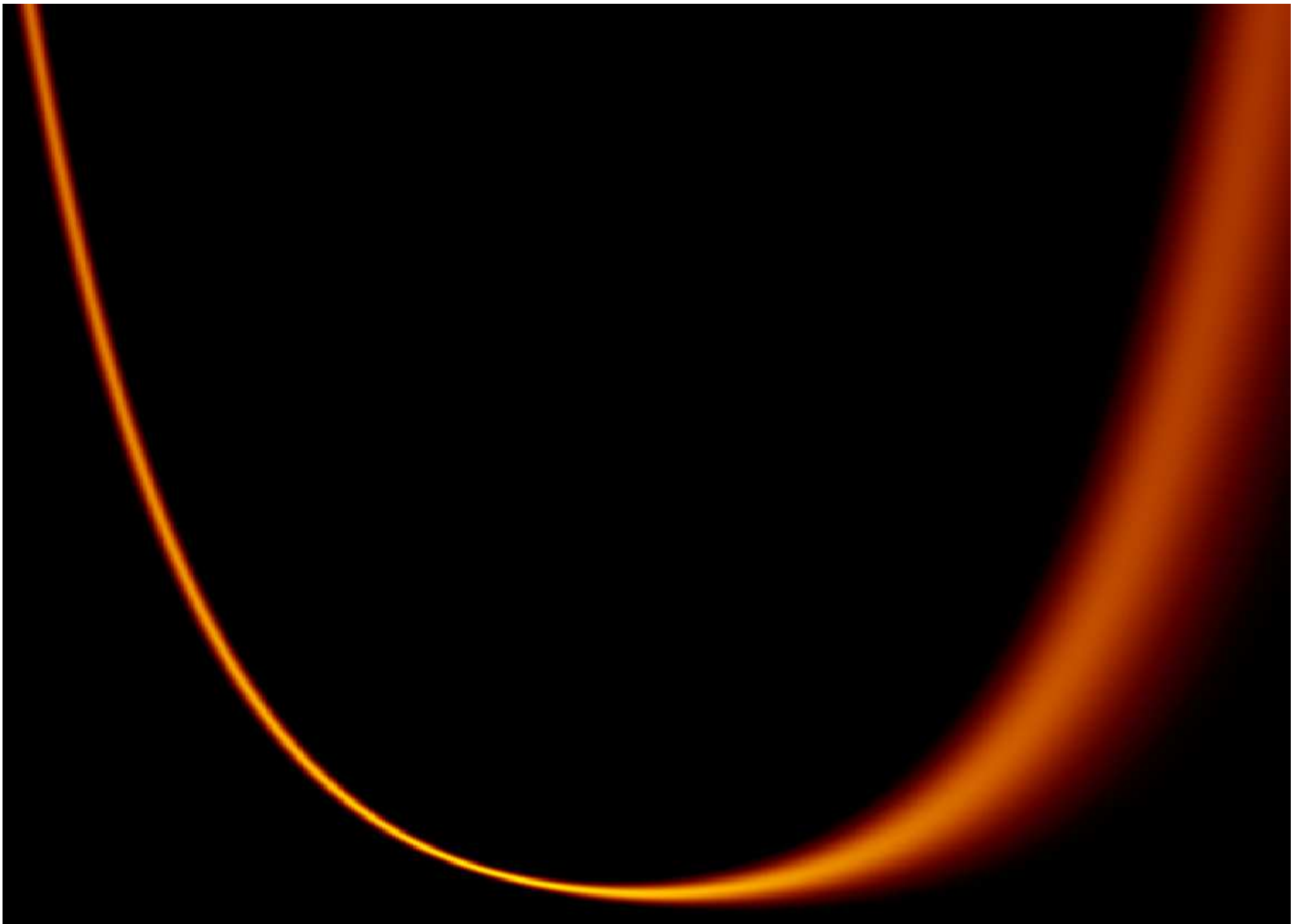
States through a bounce

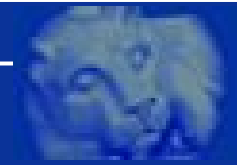
Equations of motion for expectation values decouple:

$$\frac{d\langle\hat{V}\rangle}{d\phi} = -\frac{1}{2}(\langle\hat{J}\rangle + \langle\hat{J}^\dagger\rangle) \quad , \quad \frac{d\langle\hat{J}\rangle}{d\phi} = -\frac{1}{2}(\langle\hat{V}\rangle + \hbar) = \frac{d\langle\hat{J}^\dagger\rangle}{d\phi}$$

solved by $V(\phi) = H\cosh(\phi - \delta) - \hbar$, which never reaches zero.

(Also solved by $V(\phi) = H\sinh(\phi - \delta) - \hbar$, but ruled out using physical normalization of state used for expectation value.)





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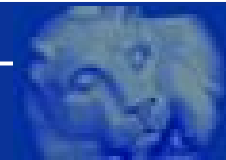
(Also solved by $V(\phi) = H\sinh(\phi - \delta) - \hbar$, but ruled out using physical normalization of state used for expectation value.)

Solvability allows determination of *dynamical coherent states* and their spreading behavior.

Can pre-bounce fluctuations be constrained well observationally?

How much can we learn about the state before the big bang (even in this special solvable model)?

Depends on assumptions we can afford to put in.

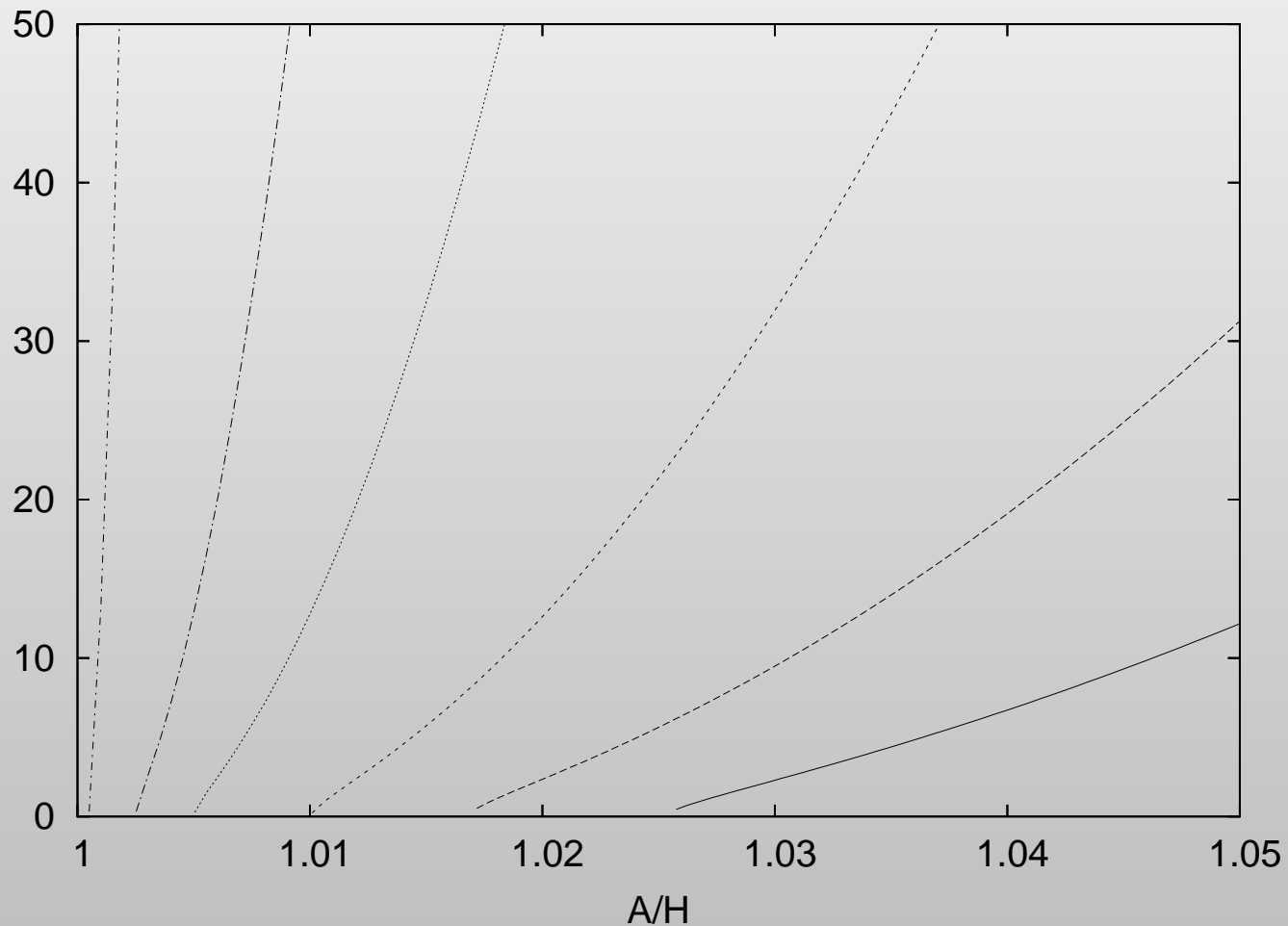


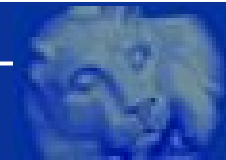
Cosmic forgetfulness

Asymmetry of fluctuations before and after the big bang extremely sensitive to initial values (A).

For increasing $H = \langle \hat{H} \rangle$, steepness increasing:

$$\left| 1 - \frac{\Delta_+}{\Delta_-} \right|$$





Effective Friedmann equation

Solvable model as basis for perturbations, e.g. when potential present. Then, fluctuations and correlations do matter:

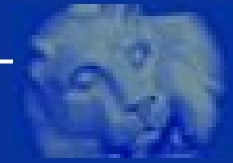
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left(\rho \left(1 - \frac{\rho_Q}{\rho_{\text{crit}}} \right) \pm \frac{1}{2} \sqrt{1 - \frac{\rho_Q}{\rho_{\text{crit}}} \eta (\rho - P) + \frac{(\rho - P)^2}{(\rho + P)^2} \eta^2} \right)$$

where P is pressure and η parameterizes quantum correlations,

$$\rho_Q := \rho + \epsilon_0 \rho_{\text{crit}} + (\rho - P) \sum_{k=0}^{\infty} \epsilon_{k+1} (\rho - P)^k / (\rho + P)^k$$

with fluctuation parameters ϵ_k ; $\rho_{\text{crit}} = 3/8\pi G \mu^2$ with scale μ .

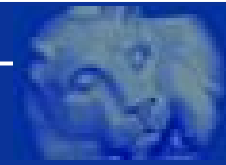
Simple behavior if $\eta = 0$ or $\rho = P$: *bounce immediate*.



Isotropic models

Interactions have several crucial effects:

- Energy density ρ_Q quantum corrected in $1 - \rho_Q/\rho_{\text{crit}}$. Would still give rise to bounce, but at different energy density. *Bounce realized for states which weakly correlated near $\rho \sim \rho_{\text{crit}}$ (e.g. semiclassical).*
- Correlations matter, add positive contribution to $(\dot{a}/a)^2$. May (or may not) prevent the bounce.
- Correlations also determine squeezing of states, and thus asymmetry between pre- and post-bounce fluctuations.



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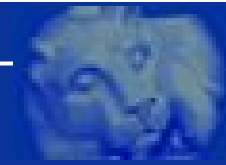
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—→ Correlations matter, add positive contribution to $(\dot{a}/a)^2$. May (or may not) prevent the bounce.

—→ Correlations also determine squeezing of states, and thus asymmetry between pre- and post-bounce fluctuations.

Crucial question is how strong *correlations* can become in high energy regime.

Note: Correlations so far mostly ignored in numerical studies.



Inhomogeneity

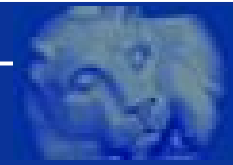
Spatial geometry subdivided if not fully homogeneous:
holonomy corrections shrink, inverse corrections grow, more
quantum back-reaction. *Generically, all corrections significant.*

Patch size ℓ_0 (coordinate length), total volume V_0 . Number of
patches: $\mathcal{N} = V_0/\ell_0^3$.

Holonomy corrections when curvature $\dot{a} \sim c_* = (\mathcal{N}/V_0)^{1/3}$.

Inverse metric corrections large when $a \sim a_* = (\mathcal{N}/V_0)^{1/3} \ell_P$.
(For $a \sim a_*$, physical patch density $\mathcal{N}/a^3 V_0 = (a_*/a)^3 / \ell_P^3$ near
one per Planck volume.)

Classical range: $\dot{a} \ll c_*$, $a \gg a_*$.



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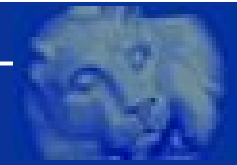
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Note: correct scaling behavior, no “gauge artefacts”
(V_0 -dependence). But only if role of \mathcal{N} recognized.

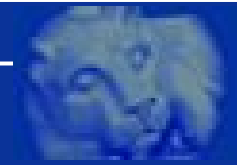


Refinement and phenomenology

\mathcal{N} may itself depend on a if discrete *structure refined during expansion*. Gives rise to different models (“ μ_0 ”: $\mathcal{N} = \text{const}$; “ $\bar{\mu}$ ”: $\mathcal{N} \propto a^3 V_0$). If power law: $\mathcal{N} \propto a^{-6x}$ with $-1/2 < x < 0$ generically according to dynamics of loop quantum gravity.

$x = 0$: No refinement, just enlarge lattice during expansion; late-time problems.

$x = -1/2$: Maximum refinement, no further excitations of spatial “atoms.”



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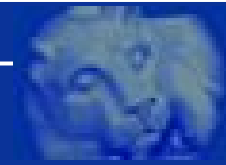
Surprisingly strong consequences! Use phenomenology to see how quantum gravity *dynamically refines its discrete space*.

Recent examples: Upper bound $\mathcal{N}/a^3 V_0 < 3/\ell_{\text{P}}^3$ from BBN.

[with R. Das, R. Scherrer]

Characteristic blue-tilt for tensor modes, enhanced if $x > -1/2$.

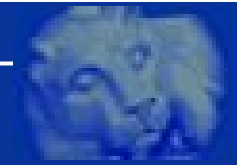
For $x = -1/2$: small correction of size $8\pi G\rho\ell_{\text{P}}^2$. [A. Barrau, J. Grain]



Covariance

Crucial for observational contact: correction terms for evolution of *gauge invariant perturbations*. Effective equations for background available, may try “classical/quantum field theory on classical/quantum background.”

But pressing *consistency*: are symmetries well-respected to ensure consistent equations of motion for gauge invariant quantities?



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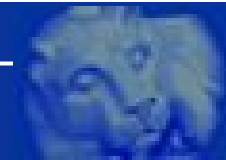
But pressing *consistency*: are symmetries well-respected to ensure consistent equations of motion for gauge invariant quantities?

Quantum commutator $[\hat{H}[N], \hat{H}[M]]$ of correct form?

Which semiclassical state for effective Hamiltonian $\langle \hat{H} \rangle$?

Can be handled with effective equations and effective constraints.

Properties of *dynamical coherent states* determined order by order in semiclassical expansion (just like interacting vacuum in perturbative quantum field theory).



Gravitational waves

Example: Propagation of gravitational waves compared to light.
Hamiltonian (for inverse volume correction)

$$H_G = \frac{1}{16\pi G} \int_{\Sigma} d^3x \alpha(E_i^a) \frac{E_j^c E_k^d}{\sqrt{|\det E|}} \left(\epsilon_i^{jkl} F_{cd}^i - 2(1 + \gamma^2) K_{[c}^j K_{d]}^k \right)$$

implies linearized wave equation

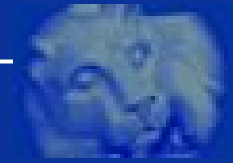
$$\frac{1}{2} \left[\frac{1}{\alpha} \ddot{h}_a^i + 2 \frac{\dot{\alpha}}{\alpha} \left(1 - \frac{2a d\alpha/da}{\alpha} \right) \dot{h}_a^i - \alpha \nabla^2 h_a^i \right] = 8\pi G \Pi_a^i$$

for tensor mode h_a^i on cosmological background with scale factor a and source-term Π_a^i .

Dispersion relation for gravitational waves:

$$\omega^2 = \alpha^2 k^2$$

$\alpha > 1$ from perturbative corrections: super-luminal?



Causality

Compare with electrodynamics, Hamiltonian:

$$H_{\text{EM}} = \int_{\Sigma} d^3x \left[\alpha_{\text{EM}}(q_{cd}) \frac{2\pi}{\sqrt{q}} E^a E^b q_{ab} + \beta_{\text{EM}}(q_{cd}) \frac{\sqrt{q}}{16\pi} F_{ab} F_{cd} q^{ac} q^{bd} \right]$$

wave equation

$$\partial_t (\alpha_{\text{EM}}^{-1} \partial_t A_a) - \beta_{\text{EM}} \nabla^2 A_a = 0$$

dispersion relation

$$\omega^2 = \alpha_{\text{EM}} \beta_{\text{EM}} k^2$$

Also “super-luminal” compared to classical speed of light.

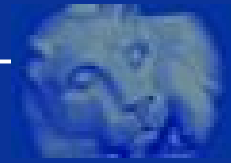
Anomaly-freedom:

$$\alpha^2 = \alpha_{\text{EM}} \beta_{\text{EM}}$$

from effective constraint algebra, physically not super-luminal.



Cosmology



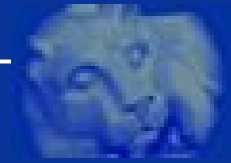
Analysis of constraints and derivation of systematic perturbation theory under development.

[with Golam Hossain, Mikhail Kagan, David Mulryne, Nelson Nunes, Juan Reyes, Subramanian Shankaranarayanan, Artur Tsobanjan]

Indirect effects of atomic space-time: small individual corrections even at high energies, can add up coherently.



Cosmology



Analysis of constraints and derivation of systematic perturbation theory under development.

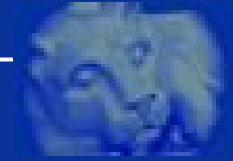
[with Golam Hossain, Mikhail Kagan, David Mulryne, Nelson Nunes, Juan Reyes, Subramanian Shankaranarayanan, Artur Tsobanjan]

Indirect effects of atomic space-time: small individual corrections even at high energies, can add up coherently.

—→ *cosmology*, high energy density, long evolution

—→ *astro-particles*, high energies from distant sources.

Different quantum effects occur and need to be disentangled.
Reliable equations for most of them will be available soon.



Conclusions

Fundamental physics of loop quantum gravity can be bridged with phenomenological scenarios.

(Weak) observational constraints already exist for the underlying discrete structure: may provide indirect evidence for atomic space.

Able to resolve problems such as singularity issue in some models, others under investigation.

Direct cosmological applications and evolution of inhomogeneities through a bounce require consistent evolution equations of observables, should be available soon.