Searching for periodic gravitational waves from neutron stars

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Abhayfest
June 4-6, 2009

Outline

- Introduction
- Periodic GWs from neutron stars
- Directed searches and the Crab result
- Wide parameter space searches
- Accreting neutron stars

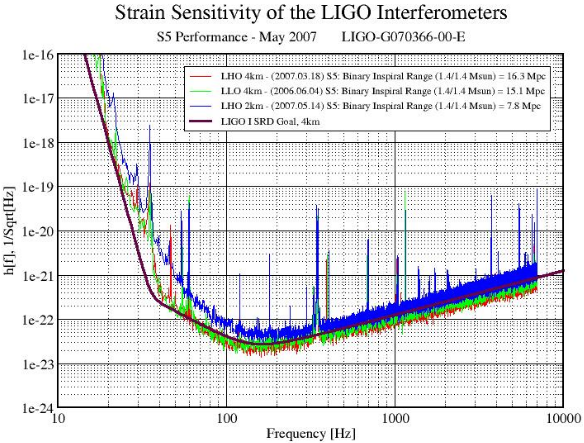
Introduction

- Classical General relativity is entering a new era where many of its predictions will be tested with astrophysical observations
- Gravitational waves are perhaps the most important tool for this
- GWs could potentially enable us to probe some of the most interesting ramifications of general relativity such as black holes
- ...and also the behavior of matter at extreme conditions as seen in neutron stars
- The first generation of kilometer-scale interferometric detectors have been built
- The LIGO detectors have reached their promised design sensitivity and Virgo is not far behind

Gravitational Wave Detectors

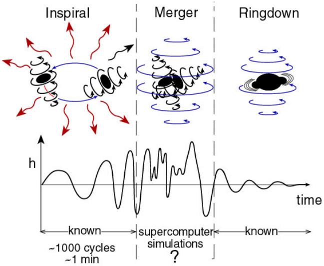
- Various ground based detectors in operation
- Bar detectors – EXPLORER, NAUTILUS, AURIGA etc.
- LSC: LIGO-Hanford, LIGO-Livingston, GEO

The LIGO detectors have reached their design goals

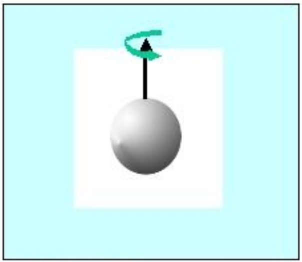


The kinds of signals expected for LIGO are

- Short duration events lasting seconds or minutes: coalescence of black holes and/or neutron stars, supernovae etc. – talk by Steve Fairhurst
- Long duration signals: stochastic back ground and periodic gravitational signals



(K. Thorne)



(T. Creighton)

The waveform

In the rest frame of the neutron star, the signal is a sinusoid with a quadrupole pattern for the amplitude:

$$\begin{aligned} h_+(\tau) &= A_+ \cos\Phi(\tau) \qquad h_\times(\tau) = A_\times \sin\Phi(\tau) \\ A_+ &= h_0 \frac{1+\cos^2\iota}{2} \qquad A_\times = h_0 \cos\iota \\ \Phi &= \Phi_0 + \int_{\tau_0}^{\tau} 2\pi\nu_{gw}(\tau')d\tau' \end{aligned} \tag{1}$$

- All emission mechanisms connect the GW frequency to the rotation of the star, e.g. $\nu_{gw} = 2\nu$ for mountains, $\nu_{gw} \approx 4\nu/3$ for r-modes etc
- The frequency evolution is determined by the dynamics
- If we had spindown due only to GW emission

$$\dot{\nu} = -\frac{G}{c^5}(\cdots)\nu^5$$

The waveform

- But reality is complicated by interactions of the star with its environment
- Magnetic dipole braking $\implies \dot{\nu} \sim \nu^3$
- This is thought to explain most of the spindown for observed pulsars
- But braking indices are not observed to be exactly 3
- More generally: $\dot{\nu} = \sum_i k_i \nu^{n_i}$
- Accreting neutron stars can be spun-up because of accretion torque
- For very rapid spindown ($\mathcal{O}(100\,\mathrm{Hz})$ in a few days) other complications might arise, e.g. the moment of inertia might change appreciably
- Neutron stars are known to glitch
- For older stars with a small spindown rate, $\nu(t)$ can be modeled as a polynomial in time

$$\nu(\tau) = \nu(\tau_0) + \dot{\nu}(\tau - \tau_0) + \frac{1}{2}\ddot{\nu}(\tau - \tau_0)^2 + \ldots$$

- This works very well for known pulsars, even over many years of observation

There are a number of proposed mechanisms for the emission

- The star might be slightly deformed from axisymmetry

$$h_0 = \frac{16\pi^2 G}{c^4} \frac{I_{zz}\epsilon f_r^2}{d}$$

($\epsilon = (I_{xx} - I_{yy})/I_{zz}$: equatorial ellipticity)

  - For a “normal” neutron star, one would expect $\epsilon \sim 10^{-7}$
  - But recent work by Horowitz & Kadau suggests these mountains could be an order of magnitude larger than previously thought
  - Other exotic forms of matter might also sustain higher deformations, e.g. could have $\epsilon \sim 10^{-3}$ for solid strange quark stars (Owen, 2005)

- Oscillation modes of the neutron star fluid, e.g. r-modes which are unstable under GW emission
- A toroidal internal magnetic could make the star prolate in shape causing it to spin down very rapidly and strongly emit GWs (Cutler, 2002)

The waveform phase

The phase model is taken to be a polynomial corresponding to a reference time τ_0:

$$\Phi(\tau) = \Phi_0 + 2\pi \left[f(\tau - \tau_0) + \frac{1}{2}\dot{f}(\tau - \tau_0)^2 + \dots \right]$$

Need to correct for the arrival times

- For an isolated pulsar:

$$\tau = t + \frac{\mathbf{r}_D \cdot \mathbf{n}}{c} + \text{relativistic corrections}$$

- For a pulsar in a binary system:

$$\tau = t + \frac{\mathbf{r}_D \cdot \mathbf{n}}{c} + \frac{\mathbf{r}_P \cdot \mathbf{n}}{c} + \text{relativistic corrections}$$

- $\mathbf{n}$: sky-position, $\mathbf{r}_D$: Detector in SSB frame, $\mathbf{r}_P$: Pulsar in binary frame
- This simple model might be complicated by glitches and accretion
- We assume the signal to last months or years

Search techniques

Fully coherent matched filter searches

- Feasible only for precisely known sources

Semi-coherent searches

- Break up data T_{obs} into N smaller segments T_{coh} and combine the segments semi-coherently
- This is forced upon us for targeted or blind searches by computational cost constraints – situation probably similar in the ET era
- Different flavors depending on what one does in the coherent and incoherent steps
- In the most general sense, this includes
  - Short coherent time baseline searches (Powerflux, Hough, Stackslide)
  - Segments are demodulated coherently ("Hierarchical search")

Search techniques

To simplify life for this talk, we write the sensitivity of the searches in two cases

- Single template search

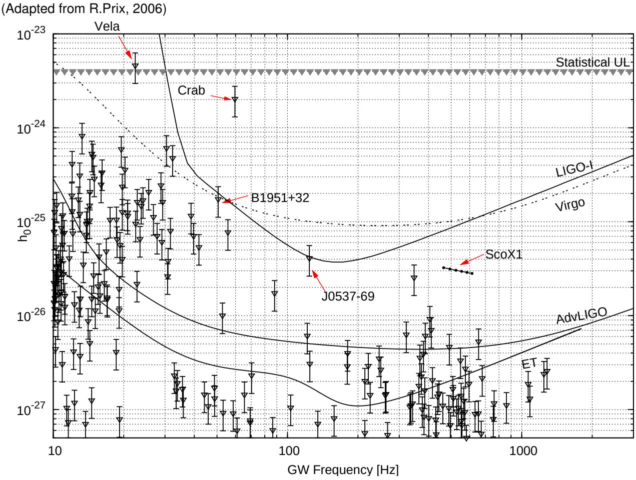
$$h_0 \approx 11 \sqrt{\frac{S_n(f)}{DT_{obs}}}$$

- Wide parameter space semi-coherent search

$$h_0 \approx \frac{25}{N^{1/4}} \sqrt{\frac{S_n(f)}{DT_{coh}}}$$

- The factor of 25 is meant to include both hits due to computational cost and multiple statistical trials
- This is just a useful fudge at the moment, and we will eventually need a more careful analysis for a given source and search technique
- Do not expect to be accurate to better than 50% with these estimates!

Targeted searches



- 1 year integration
- 3 detectors for Adv LIGO, single detector for ET and Virgo
- Error bars correspond to 10% uncertainty in distance and $I_{zz} = [1 - 3] \times 10^{38}$ kg-m^2

Summary of key LIGO results

- LIGO data has been used to do better than other indirect limits on h_0 coming mostly from EM observations
- The Crab spindown limit has been beaten: less than $\sim 6\%$ of its spindown energy is going into gravitational waves
- The spindown limit will be challenged for J0537-69 using S5 data, and Vela should be beaten by Virgo
- Indirect limits on objects like Cas A have been beaten – but this is a weaker statement than the Crab result
- The Bladford limit on h_0 based on a population of GW pulsars has been beaten by the wide parameter space semi-coherent search – though the more stringent limits by Knispen-Allen are still out of reach

- One of the most famous pulsars: The Crab (b.1054 AD)

(NASA/CXC/SAO (Chandra X-ray observatory))

- The Crab is about 2 kpc away from us, it is observed to be rotating at $\nu \approx 29.78\,\text{Hz}$
- It is spinning down at $\dot{\nu} \approx 3.7 \times 10^{-10}$ Hz/s
- This corresponds to a kinetic energy loss of $\approx 4.4 \times 10^{31}$ W (assuming $I_{zz} = 10^{38}\,\text{kg-m}^2$)
- If all of this energy loss were due to emission of gravitational waves at 2ν, then they would have an amplitude

$$h_0^{sd} = 8.06 \times 10^{-19} \frac{I_{38}}{d_{kpc}} \sqrt{\frac{|\dot{\nu}|}{\nu}} = 1.4 \times 10^{-24}$$

- In reality, most of this spindown is due to electromagnetic braking, but we want to measure this directly

- Use priors on the pulsar orientation from X-ray observations of the Nebula (Ng & Romani (2004, 2008))
- The search did not result in a detection
- The 95% degree-of-belief upper limit on h_0 with these priors is 2.7×10^{-25}
- This corresponds to an upper limit of $\approx$ 4% of the spin-down energy loss
- With uniform priors, the corresponding number is 6%
- In terms of ellipticity, this corresponds to $\epsilon := (I_{xx} - I_{yy})/I_{zz} \leq 1.8 \times 10^{-4}$
- Also performed a search in a small frequency range which led to a 95% confidence upper limit of 1.7×10^{-24} (worse due to more statistical trials)

Blind searches for isolated pulsars

- Expect to have nearby neutron stars not visible as pulsars
- Some of these might be visible in gravitational waves
- This implies a blind search in $(\nu, \dot{\nu}, \mathbf{n})$ (gr-qc/0605028)
- Sensitivity goes as $\sqrt{T_{obs}}$

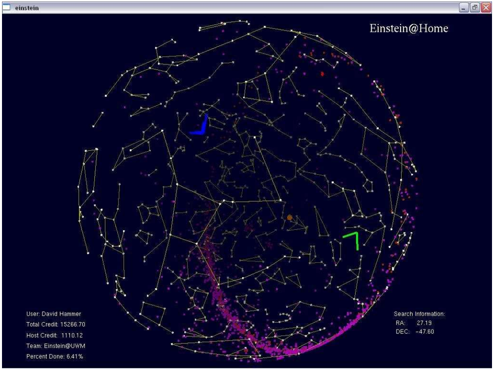
$$h_0 \propto \sqrt{\frac{S_n(f)}{T_{obs}}}$$

- Number of templates increases rapidly with T_{obs}
- For short T_{obs} ($\ll$ 1 year) we have approximately (for an all sky search including f and $\dot{f}$:

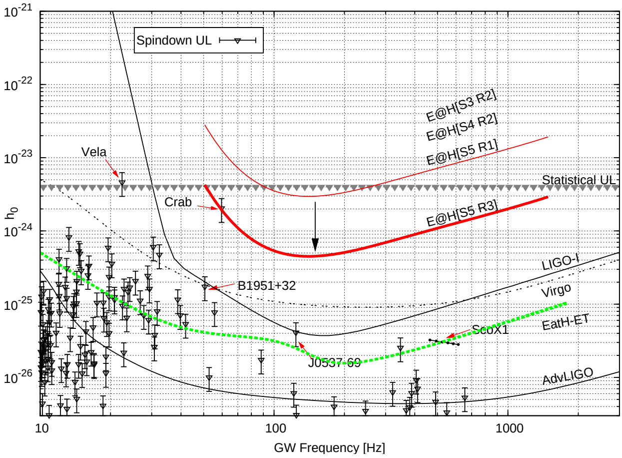
$$N_{templates} \propto T_{obs}^5$$

- And of course we need a large T_{obs} to get decent SNR
- This is a big problem...

...So we need a really big computer



- `Einstein@Home` is one of the largest projects of its kind in the world
- More than 70,000 active users currently
- provides $\sim$ 80 TFlops round the clock

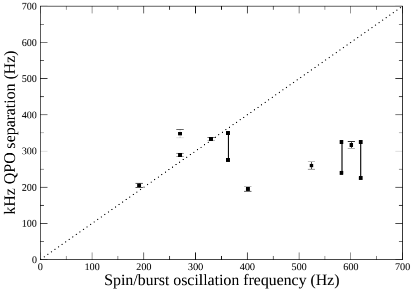


- Scale up current `Einstein@Home` search to ET sensitivity with single instrument
- Can reasonably expect to beat the spindown limit of unknown neutron stars to a few kpc

(Based on Watts, Krishnan, Bildsten & Schutz, MNRAS 2008)

- Observationally accreting NSs seem to spin slower than expected. Current record is 716 Hz and most spin much slower.
- Break-up frequency is expected to be at least 1kHz
- Three sets of sources for which we might hope to have a spin frequency measurement
  - Millisecond pulsars (10 sources)
  - Sources showing burst oscillations (MSPs with burst oscillations are consistent) (12 + 7 sources)
  - kHz QPO systems (probably weak link with spin frequency) (9 sources)

- Almost all models have some kind between kHz QPO separation $\Delta\nu_{qpo}$ and ν_s
- It was suggested that the kHz QPO separation frequency is either ν_s or $2\nu_s$
- But this link may not be real



(Hessels)

The expected GW amplitude

- Is this limit due to GW emission? (Bildsten, 1998)
- There are other possibilities involving interactions between the magnetic field and accretion disc, but we will not discuss them here
- If torques are balanced + simplifying assumptions

$$h_0^2 \propto \sqrt{\frac{R^3}{M}} \times \frac{\text{EM Flux}(\mathcal{F})}{\text{spin freq.}(\nu_s)}$$

$\Longrightarrow$ observation of $(\mathcal{F}, \nu_s)$ yields GW amplitude h_0 for this mechanism

The expected GW amplitude

- GW torque

$$\mathcal{T}_{gw} = \frac{\dot{E}_{gw}}{\Omega_s} = -\frac{32GQ^2\Omega_s^5}{5c^5}$$

- Accretion torque

$$\mathcal{T}_a = \dot{M}R^2\Omega_s = \dot{M}R^2\sqrt{\frac{GM}{R^3}} = \dot{M}\sqrt{GMR}$$

- Observed flux and luminosity

$$\mathcal{F} = \frac{L}{4\pi d^2} \quad L = \frac{GM\dot{M}}{R} \implies \dot{M} = \frac{4\pi R d^2 \mathcal{F}}{GM}$$

- The GW frequency: $\nu_{gw} = 2\nu_s$ (mountain) or $\nu_{gw} = 4\nu_s/3$ (r-modes)

- The GW amplitude for the mountain scenario

$$h_0^2 = \frac{5G}{2\pi^2 c^3 d^2 \nu_{gw}^2} \dot{E}_{gw} = \frac{5G}{2\pi^2 c^3 d^2 \nu_{gw}^2} \mathcal{T}_a \Omega_s \propto \frac{\mathcal{F}}{\nu_s}$$

- Putting all the numbers together:

$$h_0 = 3 \times 10^{-27} F_{-8}^{1/2} \left(\frac{R}{10\,\mathrm{km}} \right)^{3/4} \left(\frac{1.4 M_\odot}{M} \right)^{1/4} \left(\frac{1\,\mathrm{kHz}}{\nu_s} \right)^{1/2}$$

where $F_{-8} := \mathcal{F}/10^{-8} \mathrm{erg\ cm}^{-2} \mathrm{s}^{-1}$

- The best case detectable amplitude for D detectors with PSD S_n and observation time T_{obs} is (FA = 0.01, FD = 0.1)

$$h_0 = 11.4 \sqrt{\frac{S_n(\nu_{gw})}{D T_{obs}}}$$

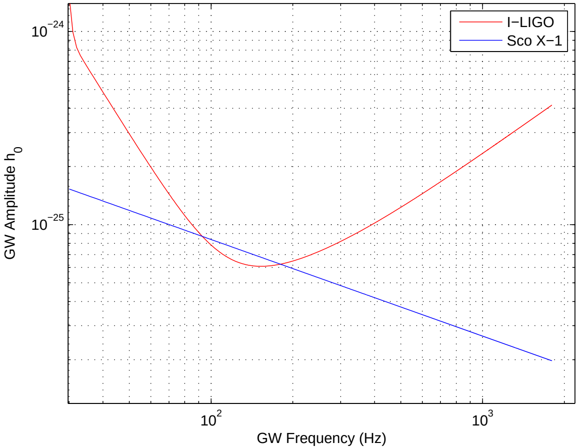
Astrophysical payoffs of a detection

- GW frequency will determine emission mechanism
- GW amplitude will determine degree of mass asymmetry or velocity field leading to constraints on NS interiors
- Will lead to constraints on accretion disk models and NS magnetic field
- Sufficiently tight upper limits could rule out GW balance mechanism

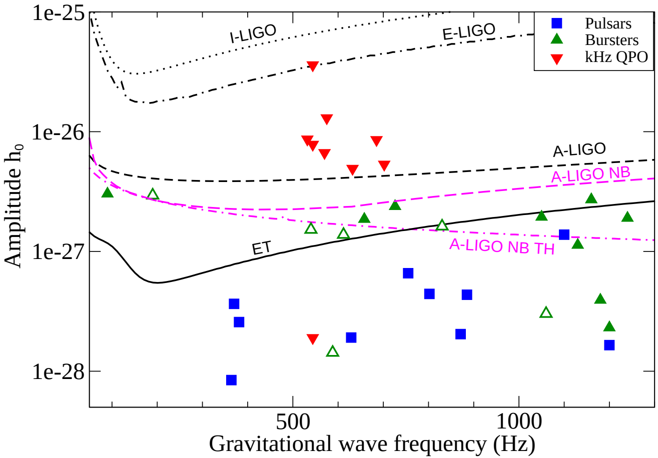
Sco X-1 could be detectable with initial LIGO

- Sco X-1 is the brightest LMXB ($P_{orb} = 18.895\,\text{h}$, $a_x \sin\iota = 1.44\,\text{lt-sec}$, $d = 2.8\,\text{kpc}$)

Expected and observable GW amplitudes for Sco X-1:



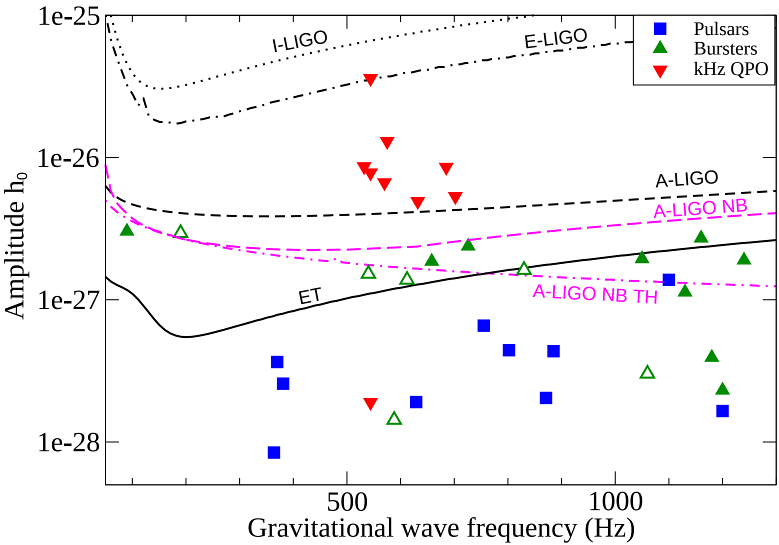
(From Watts et al., 2008)



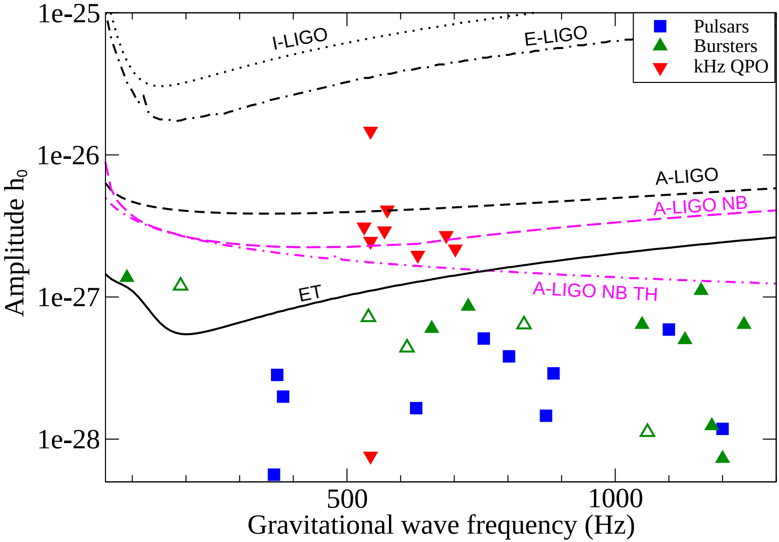
- 2 year integration, single template
- Assume frequency is known for kHz QPO sources
- Very important to have X-ray timing missions in ET era!

- Searches for binary systems till now have not even been close to making a detection by at best 2 orders of magnitude
- Best case scenarios show that detection is possible with Advanced LIGO
- The real search involves a large parameter space. This degrades the sensitivity in two ways
  - The computational cost might restrict T_{obs} so that the full data set might not be usable
  - There will be statistically more false alarms for the same threshold causing the effective threshold to be raised
- The first effect is more important but both need to be taken into account

Effect of the statistical and computational hits



Effect of the statistical and computational hits



Outlook

- We don't yet have a detection yet but we are getting there
- The aim is not just detection, but to do gravitational wave astronomy
- Neutron stars might emit detectable periodic GWs
- A significant effort is devoted to detect these signals
- A detection would give us information about matter at extreme conditions and how neutron stars interact with their surroundings