A Holographic Description of Black Hole Singularities

Gary Horowitz
UC Santa Barbara

IGPG04/8-4, AEI–2004–072

Black hole evaporation: A paradigm

Abhay Ashtekar[1] and Martin Bojowald[2,1]

1. Institute for Gravitational Physics and Geometry,
Physics Department, Penn State, University Park, PA 16802, USA

2. Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut,
Am Mühlenberg 1, D-14476 Potsdam, Germany

Abstract

A paradigm describing black hole evaporation in non-perturbative quantum gravity is developed by combining two sets of detailed results: i) resolution of the Schwarzschild singularity using quantum geometry methods [1, 2]; and ii) time-evolution of black holes in the trapping and dynamical horizon frameworks [3, 4, 5, 6]. Quantum geometry effects introduce a major modification in the traditional space-time diagram of black hole evaporation, providing a possible mechanism for recovery of information that is classically lost in the process of black hole formation. The paradigm is developed directly in the Lorentzian regime and necessary conditions for its viability are discussed. If these conditions are met, much of the tension between expectations based on space-time geometry and structure of quantum theory would be resolved.

PACS numbers: 0460P, 0470D

Global event horizons do not exist in quantum gravity:

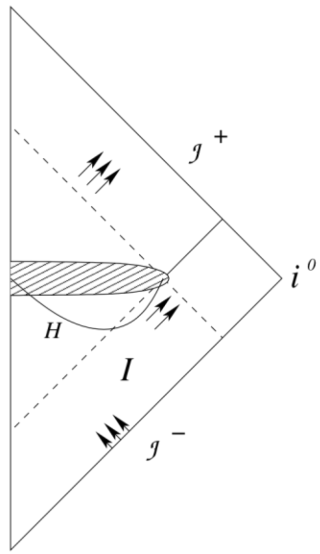


FIG. 2: Space-time diagram of black hole evaporation where the classical singularity is resolved by quantum geometry effects. The shaded region lies in the 'deep Planck regime' where geometry is genuinely quantum mechanical. H is the trapping horizon which is first space-like (i.e., a dynamical horizon) and grows because of infalling matter and then becomes time-like (i.e., a time-like membrane) and shrinks because of Hawking evaporation. In region I, there is a well-defined semi-classical geometry.

String theory predicts that quantum gravity is holographic:

Physics in a region is completely described by fundamental degrees of freedom living on the boundary.

AdS/CFT Correspondence

(Maldacena, 1997)

AdS: Anti de Sitter spacetime

CFT: Ordinary (nongravitational) quantum field theory that is conformally invariant.

The AdS/CFT correspondence states that string theory on spacetimes that asymptotically approach AdS x K is completely equivalent to a CFT living on the boundary.

Advantages of using AdS/CFT:

Maps the problem of spacetime singularities into a problem in ordinary field theory

Disadvantages of using AdS/CFT:

a)The world is not asymptotically AdS
b)It has been difficult to describe observers falling into a black hole in the CFT

Outline

I. Simple example:
   A. Bulk spacetime
   B. Dual CFT
II. Implications for singularities
III. Generalizations
IV. Conclusions

(based on A. Lawrence, G. H., and E. Silverstein, arXiv:0904.3922)

D-brane tutorial

- D-branes are extended objects in 10D string theory.
- They carry a charge, and there is no force between two D-branes.
- Open strings end on D-branes.
- A stack of N D3-branes has a near horizon geometry $AdS_5 \times S^5$.

Simple example

Consider the following static black hole

$$ds^2 = -\left(r^2 - 1\right) dt^2 + \frac{dr^2}{r^2 - 1} + r^2 d\sigma^2$$

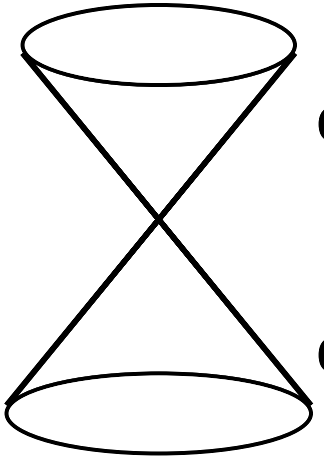
where $d\sigma^2$ is the metric on a unit 3D hyperboloid, compactified to finite volume.

This metric is locally equivalent to AdS_5. It is a higher dimensional analog of the 3D BTZ black hole.

In Minkowski spacetime, the metric inside the light cone, in Milne coordinates, is:

$$ds^2 = - dt^2 + t^2\, d\sigma^2$$

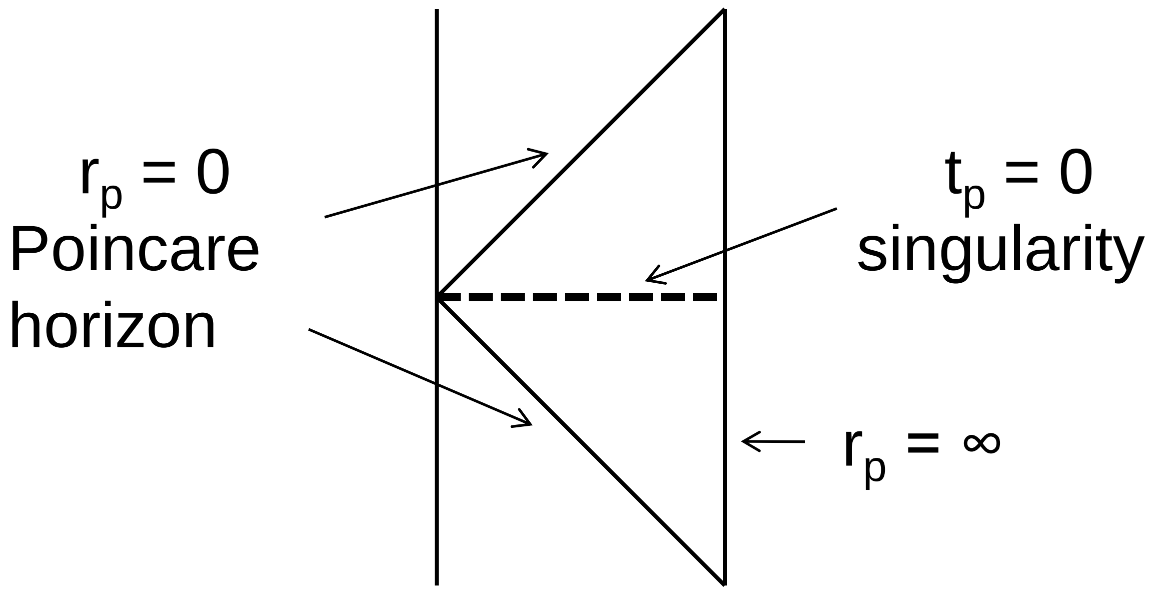
One can identify points so that $d\sigma^2$ becomes compact.



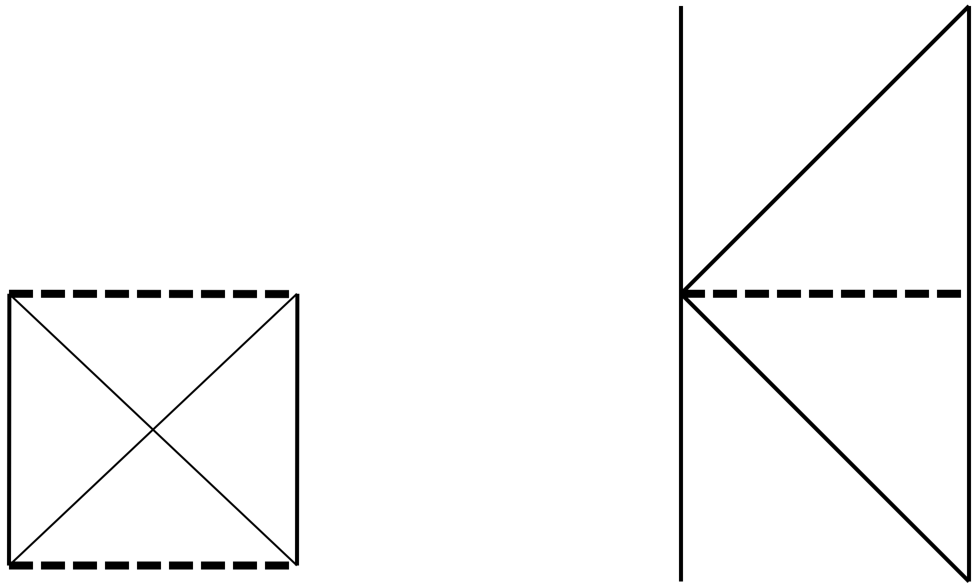
AdS can be written in Poincare coordinates

$$ds^2 = r_p^2(-dt_p^2 + t_p^2 d\sigma^2) + \frac{dr_p^2}{r_p^2}$$

One can make a similar identification on each Minkowski slice.



What is the relation between the black hole and the Poincare patch?



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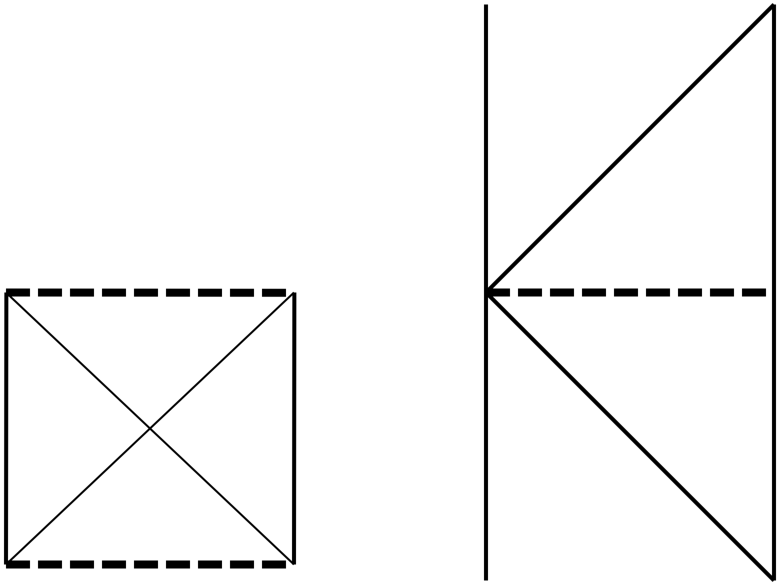

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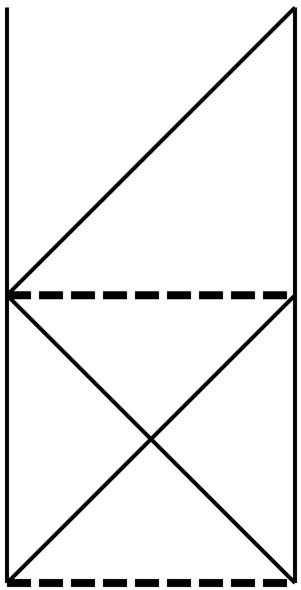
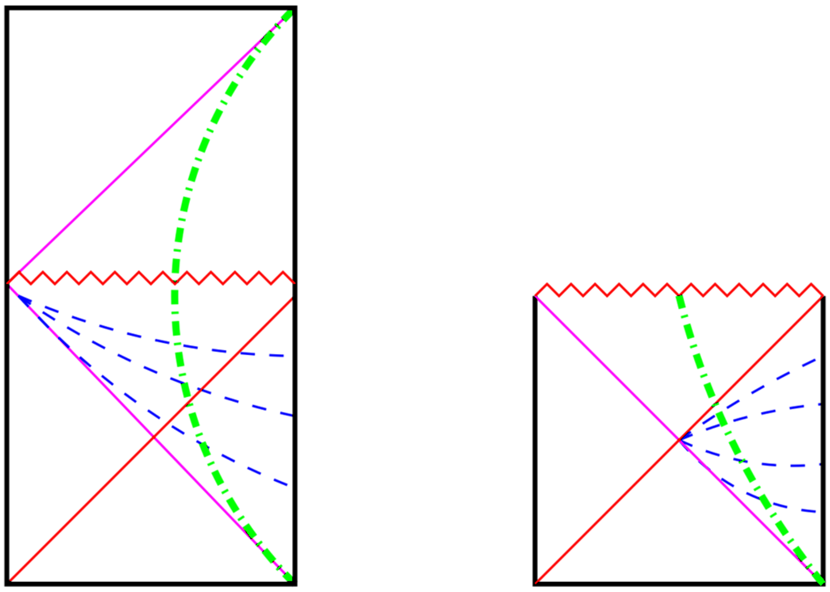


Since we know how to describe physics in the Poincare patch, we can describe physics inside the horizon.

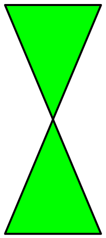
Moreover, we can easily describe infalling observers, since a D-brane stays at constant Poincare radius and this crosses the black hole horizon.



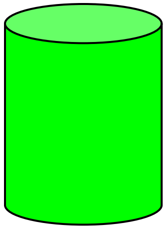
(A) (B)

Green line is motion of a D-brane
Blue lines are
A)Poincare time slices
B)Schwarzschild time slices

The natural metric on the boundary at infinity in the Poincare coordinates is the cone.



The natural metric in the black hole coordinates is a static cylinder.



These are related by a conformal transformation:

$$(1/t^2)\ [-\ dt^2 + t^2\ d\sigma^2] = -\ d\eta^2 + d\sigma^2 \qquad t = \pm\ e^{\pm\eta}$$

The collapsing and expanding cone each become an infinite static cylinder.

Metric in Poincare coordinates:

$$ds^2 = r_p^2(-dt_p^2 + t_p^2 d\sigma^2) + \frac{dr_p^2}{r_p^2}$$

Metric in Schwarzschild coordinates:

$$ds^2 = -\left(r^2 - 1\right) dt^2 + \frac{dr^2}{r^2 - 1} + r^2 d\sigma^2$$

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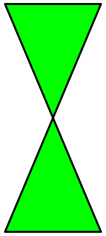

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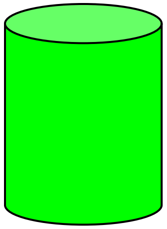

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$$(1/t^2)\,[-\,dt^2 + t^2\,d\sigma^2] = -\,d\eta^2 + d\sigma^2 \qquad t = \pm\,e^{\pm\eta}$$

The collapsing and expanding cone each become an infinite static cylinder.

Dual CFT description

If the bulk spacetime is asymptotically $AdS_5 \times S^5$, the dual CFT is U(N) super Yang-Mills (SYM).

In the Poincare patch, the SYM naturally lives on the collapsing cone.

This describes physics inside the horizon before the singularity is reached.

SYM has six scalars which are N x N matrices.

The static D-brane in Poincare coordinates is described by setting one of the scalar eigenvalues to a constant $\phi = \phi_0$.

This constant value corresponds to the radial position of the brane: $\phi_0 = r_p$.

Description in terms of SYM on static cylinder:

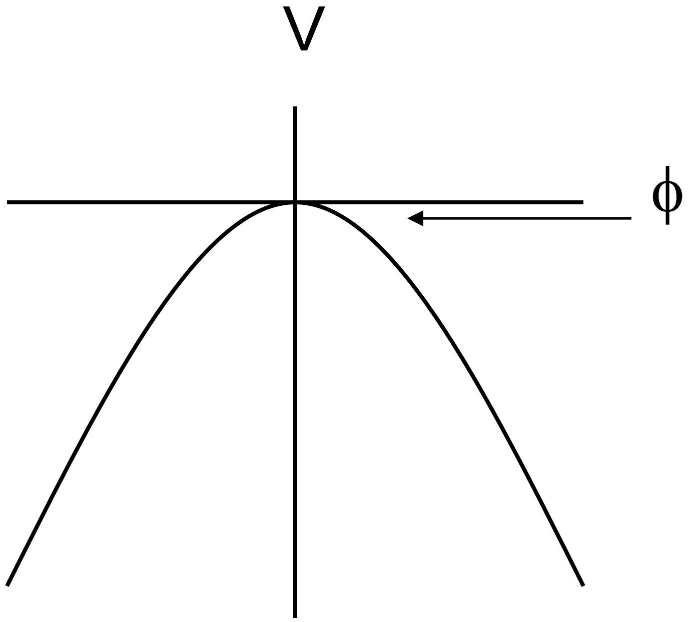
The SYM scalars couple to the curvature of space via $R\phi^2$. The static cylinder:

$$ds^2 = - d\eta^2 + d\sigma^2$$

has negative curvature, so the scalars feel a potential $V(\phi) = - \phi^2$. The solution $\phi = 0$ is unstable.

Note: In some cases, only the zero mode of ϕ is unstable. Inhomogeneous modes have $m^2_{eff} > 0$.

Under conformal transformation from cone to static cylinder, $\phi_s = t_p\, \phi_p$. The solution $\phi = \phi_0$ on the collapsing cone corresponds to $\phi = \phi_0\, e^{-\eta}$ on the static cylinder.



Equating the area of the hyperbolic space in Poincare coordinates and black hole coordinates yields $r = t_p\, r_p$, since

$$ds^2 = r_p^2(-dt_p^2 + t_p^2 d\sigma^2) + \frac{dr_p^2}{r_p^2}$$

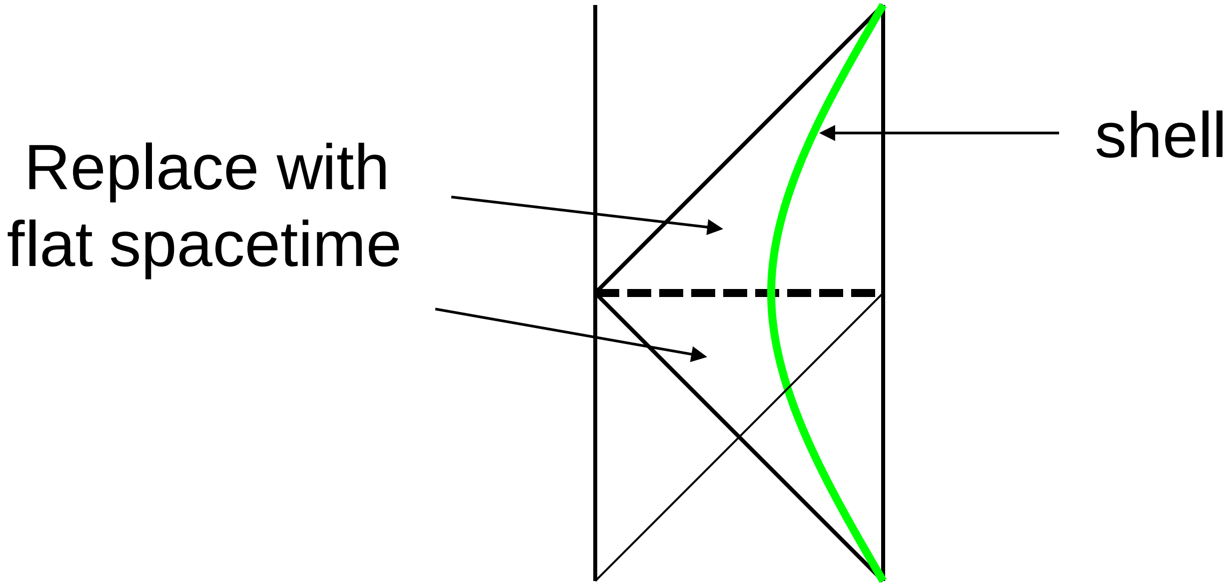
We know $\phi_p = r_p$, so $\phi_s = t_p\, \phi_p = r$.

The scalar field again gives the radial position of the D-brane even in Schwarzschild coordinates. The singularity corresponds to $\phi_s = 0$.

II. Implications for the singularity

Consider a static spherical shell of D-branes (in Poincare coordinates).

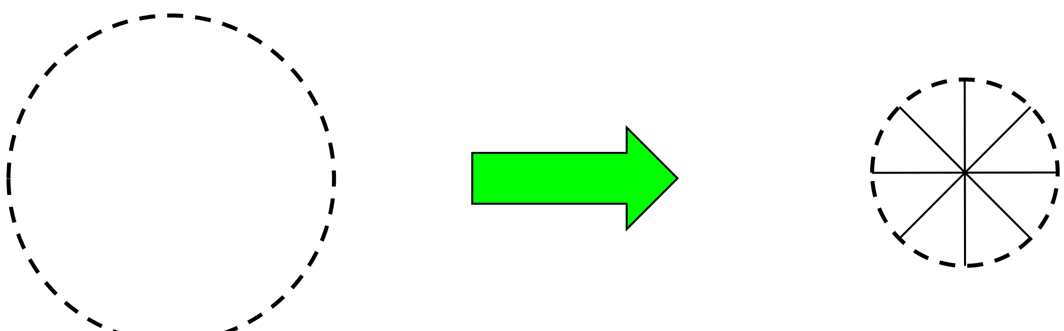
In the black hole interpretation, the shell collapses to form the hyperbolic black hole.



Initially, the SYM scalars are diagonal with eigenvalues coming in from infinity. The off diagonal modes are very massive. As the eigenvalues approach zero, the off diagonal modes become excited. The eigenvalues are trapped near zero. (Kofman et. al., 2004)

Spacetime picture:

Large shell of D-branes



Open strings excited

Since SYM is strongly coupled, you produce a complicated excited state involving all N^2 degrees of freedom.

Locality probably breaks down:

Away from the singularity, locality can be measured by scalar eigenvalues.

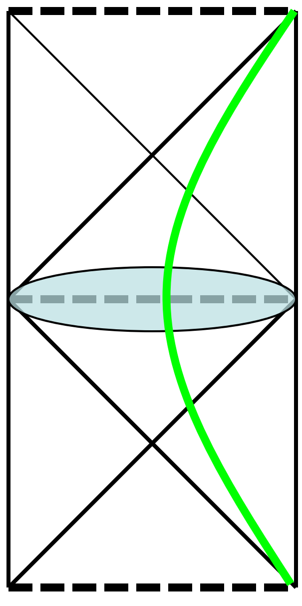
Near the singularity, all of the eigenvalues interact strongly with off-diagonal modes and with each other, the D-brane probes are no longer good definitions of any geometry.

If N is infinite, the eigenvalues will be trapped forever. This describes the formation of a classical black hole.

If N is large but finite, eigenvalues will be trapped for a time $T \sim e^{cN}$.
This is Hawking evaporation of D-branes from the black hole.

(Finite N means quantum gravity important. Hyperbolic black holes have positive specific heat. Only evaporate by emitting D-branes.)

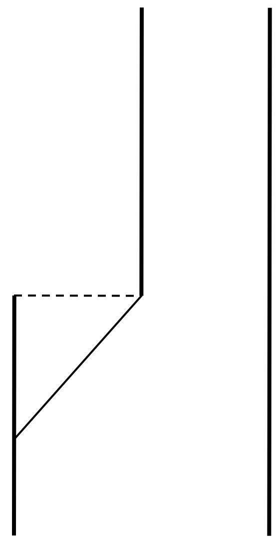
What is final spacetime picture? It is NOT just a smoothing out of the spacetime near the singularity.



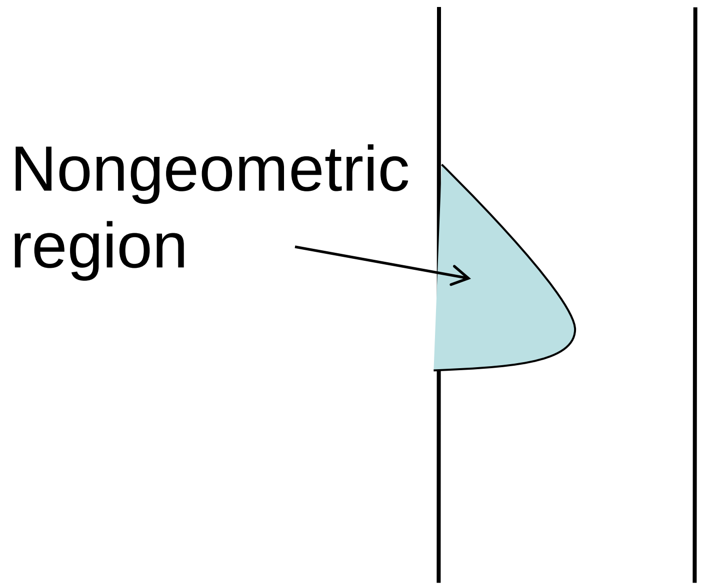
not correct

The branes come out in finite time in the SYM on the cylinder.

The branes emerge randomly, not as a coherent shell.



Standard picture of evaporating black hole in AdS



Picture motivated by dual field theory

Key lesson

Event horizons don't exist in quantum gravity

(Just like Ashtekar and Bojowald said!)

Event horizons require global causal relations which are not defined in spacetimes with nongeometric regions. (Trapped surfaces and apparent horizons will still exist.)

III. Generalizations

A hyperbolic black hole can have different masses: (Emparan)

$$ds^2 = -\left(r^2 - 1 - \frac{\mu}{r^2}\right) dt^2 + \frac{dr^2}{r^2 - 1 - \frac{\mu}{r^2}} + r^2 d\sigma^2$$

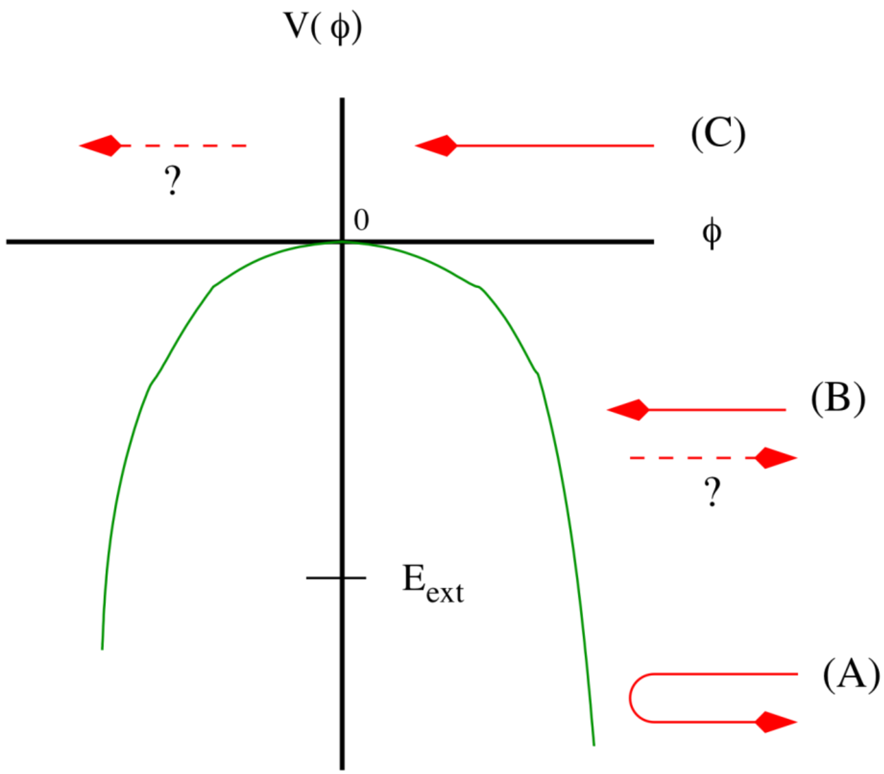
There are three cases:

$\mu > 0$: black hole with spacelike singularity

$-1/4 < \mu < 0$: black hole with timelike singularity

$\mu < -1/4$: naked singularity

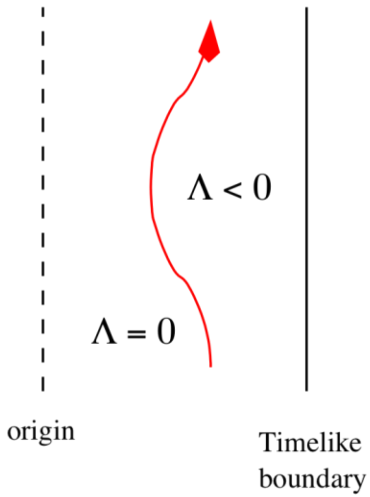
These cases are correlated with the possible motion of the scalars:



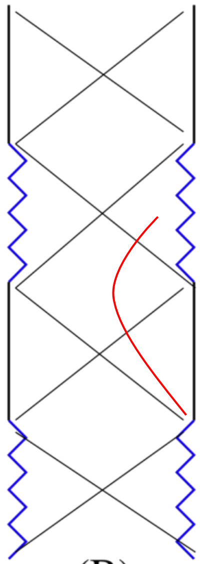
For $E > E_{ext}$ classical evolution is modified by quantum corrections

$E = E_{ext}$ corresponds to $\mu = -1/4$

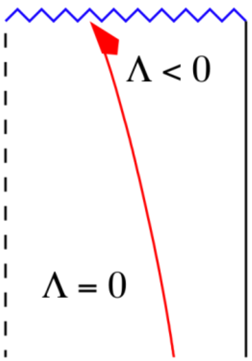
Motion of shell in classical spacetimes



(A)



(B)



(C)

Can’t form a naked singularity since the shell bounces

This case is currently under study

Conclusions

- Can describe formation of a hyperbolic black hole by collapsing a shell of D-branes.
- The physics near the singularity is governed by the SYM with small ϕ. The problem of singularities is no longer that the theory breaks down but simply that it is hard to calculate.
- The event horizon is not well defined in the quantum theory.
- The qualitative behavior of hyperbolic black holes with different mass is correlated with behavior of the scalars in the field theory.

Happy 60th Birthday Abhay!