BLACK HOLE ENTROPY

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Work of many people, including: **Ashtekar**, *Baez, Barbero, AC, Domagala, Jacobson, Lewandowski, Meissner, Engle, Fernandez-Borja, Diaz-Polo, Krasnov, Kaul, Ghosh, Majumdar, Mitra, Perez, Rovelli, Sahlmann, Villaseñor and more ...*

BLACK HOLES AND QUANTUM GRAVITY ?

Black Holes are, as Chandrasekhar used to say: *"... the most perfect objects there are in The Universe: the only elements in their construction are our concepts of space and time. Since GR predicts a single family of solutions, they are the simplest as well."* **They are the crown of classical physics in terms of their simplicity and beauty.**

But, Bekenstein and Hawking told us that :

i) Black Holes satisfy some 'thermodynamic-like laws'.

$$\delta M = \frac{\kappa}{8\pi G}\,\delta A \quad \Rightarrow \quad M \leftrightarrow E, \quad \kappa \leftrightarrow T\,, \quad A \leftrightarrow S$$

ii) When one invokes quantum mechanics ($\hbar$) then something weird happens:

$$E = M$$

$$T = \frac{\kappa\,\hbar}{2\pi}\ ,$$

and

$$S = \frac{A}{4\,G\hbar}$$

Black holes seem to have thermodynamic properties! What are then the underlying degrees of freedom responsible for entropy?

The standard wisdom is that only with a full marriage of the Quantum and Gravity will we be able to understand this.

Different approaches:

- **String Theory**
- **Causal Sets**
- **Entanglement Entropy**
- **Loop Quantum Gravity (This talk)**

QUESTIONS TO BE ADDRESSED

- **How do we characterize black holes in equilibrium? (Engle's talk)**
- **Can we define quantum horizon states?**
- **Which states should we count?**
- **How does the entropy behave?**
- **Large BH: Bekenstein-Hawking entropy**

The Beginning

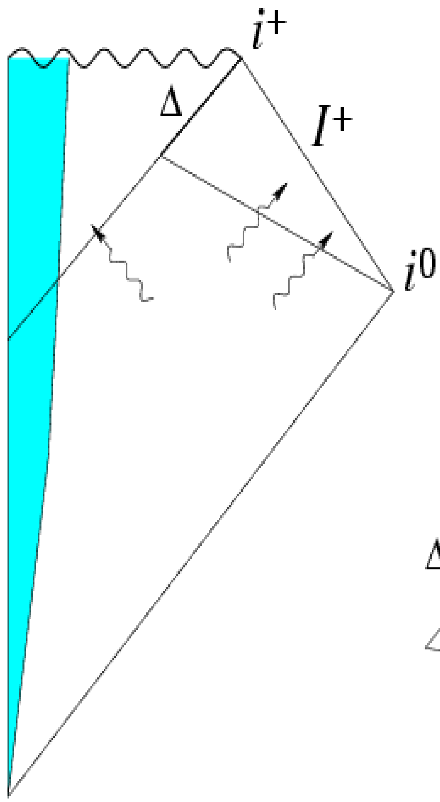
Physically, one is interested in describing black holes in equilibrium. That is, equilibrium of the horizon, not the exterior. Can one capture that notion via boundary conditions?

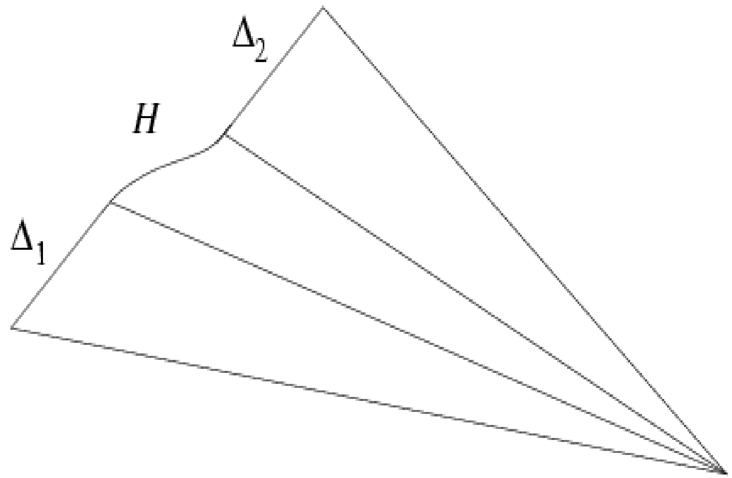
Yes! Answer: Isolated Horizons

Isolated horizon boundary conditions are imposed on an inner boundary of the region under consideration.

The interior of the horizon is cut out. In this a physical boundary?
No! but one can ask whether one can make sense of it:

What is then the physical interpretation of the boundary?





• The null boundary Δ, the 3-D isolated horizon, provides an effective description of the degrees of freedom of the inside region, that is cut out in the formalism.

- **The boundary conditions are such that they capture the intuitive description of a horizon in equilibrium and allow for a consistent variational principle.**

- **One can use loop quantum geometry in the bulk and include the boundary.**

- **The quantum geometry of the horizon has independent degrees of freedom that fluctuate 'in tandem' with the bulk quantum geometry.**

- **The quantum boundary degrees of freedom are then responsible for the entropy.**

- **The entropy thus found can be interpreted as the entropy assigned by an 'outside observer' to the (2-dim) horizon $S = \Sigma \cap \Delta$.**

- **INTERPRETATIONAL ISSUES:**

Is this to be regarded as the entropy contained by the horizon?

Is there some 'holographic principle' in action?

Can the result be associated to entanglement entropy between the interior and the exterior?, etc.

How restrictive is the condition that the black hole horizon is assumed to be there from the beginning?

ISOLATED HORIZONS (Quick Reminder)

An isolated horizon is a null, non-expanding horizon Δ with some notion of translational symmetry along its generators. There are two main consequences of the boundary conditions:

- **The gravitational degrees of freedom induced on the horizon are captured in a $U(1)$ connection,**
  $W_a = -\frac{1}{2}\,\Gamma^i_a\, r_i$

- **The total symplectic structure of the theory (and this is true even when matter is present) gets split as,** $\Omega_{\rm tot} = \Omega_{\rm bulk} + \Omega_{\rm hor}$ **with**

$$\Omega_{\rm hor} = \frac{a_0}{8\pi^2\, G\gamma} \oint_S {\rm d}W \wedge {\rm d}W'$$

- **The 'connection part' and the 'triad part' at the horizon S must satisfy the condition,** $F_{ab} = -\frac{2\pi\,\gamma}{a_0}\, E^i_{ab}\, r_i$, **the 'horizon constraint'.**

CONSTRAINTS

The formalism tells us what is gauge and what not. In particular, with regard to the gravitational constraints we know that:

• The relation between curvature and triad, the horizon constraint, is equivalent to Gauss' law.

• Diffeomorphisms that leave S invariant are gauge (vector field are tangent to S).

• The scalar constraint must have a vanishing shift $N|_{\rm hor} = 0$ on the horizon. Thus, the scalar constraint leaves the horizon untouched; any gauge and diff-invariant observable *is* a Dirac observable. For instance, all multipole moments.

In the quantum theory of the horizon we have to implement these facts.

QUANTUM THEORY: THE BULK (See Thiemann's talk)

A canonical description in terms of $SU(2)$ Yang-Mills:

$$A_a^i \quad SU(2)\,\text{connection} \quad ; \quad E_i^a \quad \text{triad}$$

with $A_a^i = \Gamma_a^i - \gamma\, K_a^i$. Loop Quantum gravity on a manifold without boundary is based on two fundamental observables of the fundamental variables :

Holonomies, $h_e(A) := \mathcal{P}\exp(\int_e A)$

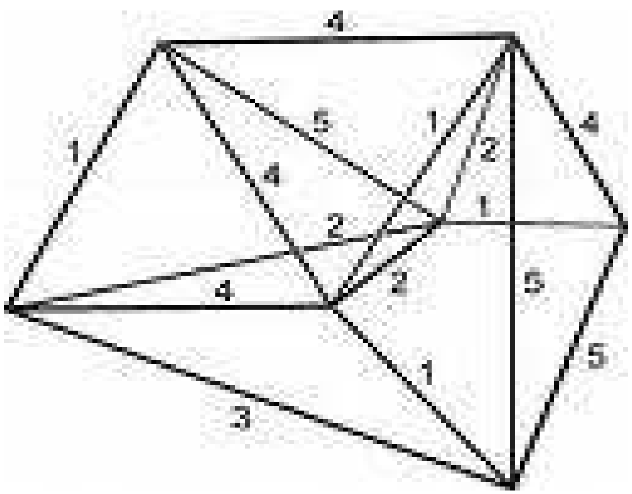
and

Electric Fluxes, $E(f, S) := \int_S \mathrm{d}S^{ab} E_{ab}^i\, f^i$

The main assumption of Loop Quantum Gravity is that these quantities become well defined operators. (LOST Theorem: There is a unique representation on a Hilbert space of these observables that is *diffeomorphism invariant*).

Hilbert space:

$$\mathcal{H}_{\mathrm{AL}} = \oplus_{\mathrm{graphs}} \mathcal{H}_{\Upsilon} = \text{Span of all Spin Networks } |\Upsilon, \vec{j}, \vec{m}\rangle \tag{1}$$

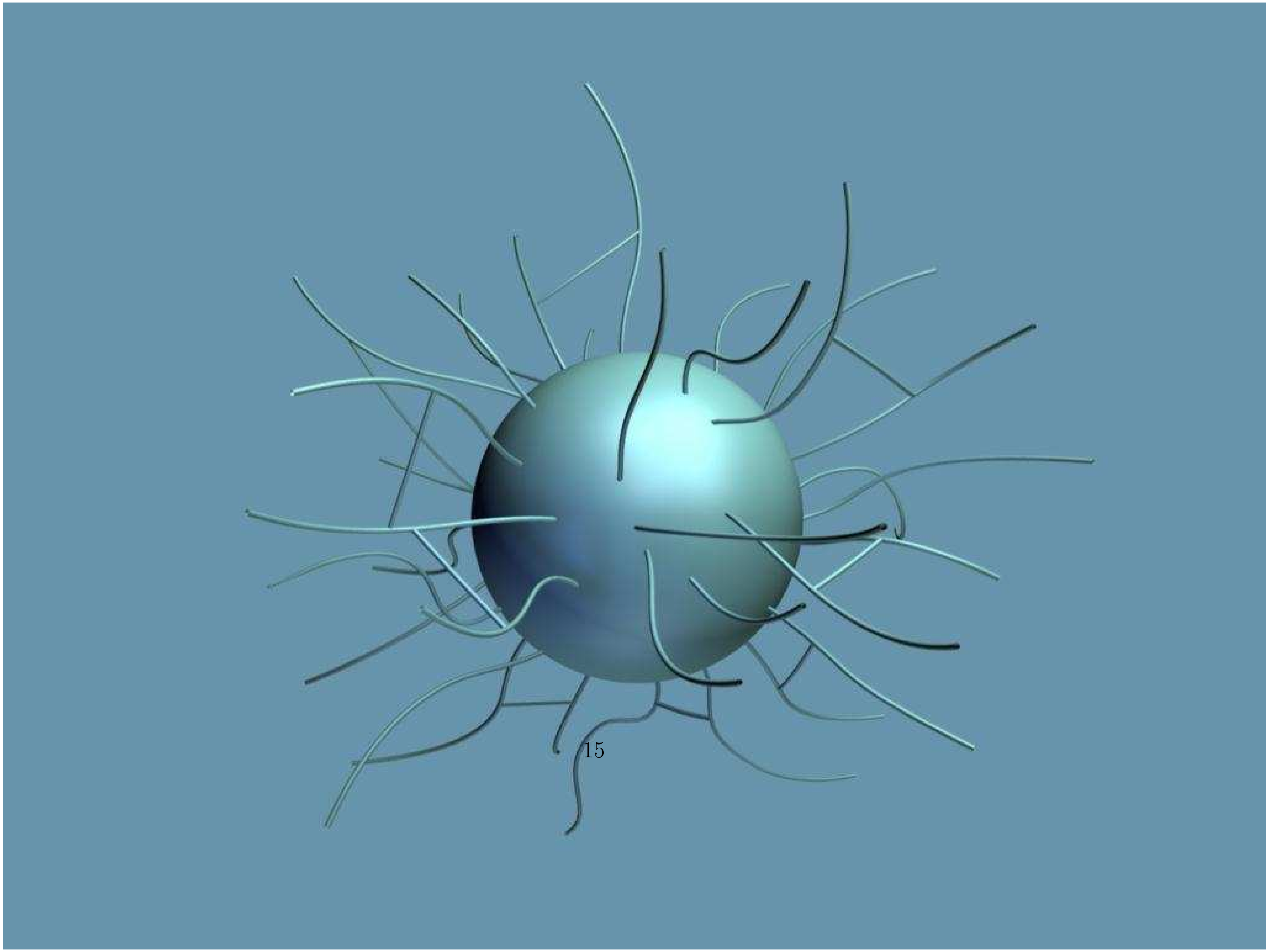


A Spin Network $|\Upsilon, \vec{j}, \vec{m}\rangle$ is a state labeled by a graph Υ, and some colorings $(\vec{j}, \vec{m})$ associated to edges and vertices.

The spin networks have a very convenient interpretation. They are the eigenstates of the quantized geometry, such as the area operator,

$$\hat{A}[S] \cdot |\Upsilon, \vec{j}, \vec{m}\rangle = 8\pi\ell_{\rm Pl}^2\gamma \sum_{\rm edges} \sqrt{j_i(j_i+2)}\, |\Upsilon, \vec{j}, \vec{m}\rangle \tag{2}$$

One sees that the edges of the graph, excite the quantum geometry of the surface S at the intersection points between S and Υ.



HORIZON QUANTUM THEORY

Total Hilbert Space is of the form:

$$\mathcal{H} = \mathcal{H}_{\rm V} \otimes \mathcal{H}_{\rm S}$$

where $\mathcal{H}_{\rm S}$, the surface Hilbert Space, can be built from Chern Simons Hilbert spaces for a sphere with punctures.

The conditions on $\mathcal{H}$ that we need to impose are: Invariance under diffeomorphisms of S and the quantum condition on Ψ:

$$\left(\mathrm{Id} \otimes \hat{F}_{ab} + \frac{2\pi\,\gamma}{a_0}\,\hat{E}^i_{ab}\,r_i \otimes \mathrm{Id} \right) \cdot \Psi = 0$$

Then, the theory we are considering is a quantum gravity theory, with an isolated horizon of fixed area a_0 (and other multiple-moments). Physical state would be such that, in the bulk satisfy the ordinary constraints and, at the horizon, the quantum horizon condition.

ENTROPY

We are given a black hole of area a_0. What entropy can we assign to it? Let us take the microcanonical viewpoint. We shall count the number of horizon states $\mathcal{N}$ such that they are compatible with the macroscopic constraints and satisfy:

- **The area eigenvalue $\langle \hat{A} \rangle \in [a_0 - \delta, a_0 + \delta]$**
- **The quantum horizon condition.**

The entropy $\mathcal{S}$ will be then given by

$$\mathcal{S} = \ln \mathcal{N}.$$

The challenge now is to identify those states that satisfy the two conditions, and count them.

CHARACTERIZATION OF THE STATES

There is a convenient way of characterizing the states by means of the spin network basis. If an edge of a spin network with label j_i ends at the horizon S, it creates a puncture, with label j_i. The area of the horizon will be the area that the operator on the bulk assigns to it: $A = 8\pi\gamma\ell_{\rm Pl}^2 \sum_i \sqrt{j_i(j_i+1)}$.

Is there any other quantum number associated to the punctures p_i? Yes! the eigenstates of $\hat{E}_{ab}$ that are also half integers m_i, such that $-|j_i| \leq m_i \leq |j_i|$. The quantum horizon condition relates these eigenstates to those of the horizon Chern-Simons theory. The requirement that the horizon is a sphere (topological) then imposes a 'total projection condition' on $m's$:

$$\sum_i m_i = 0$$

A 'configuration' of the quantum horizon is then characterized by a set of punctures p_i and to each one a pair of half integer (j_i, m_i).

The counting has three steps:

i) Given the classical area a_0, find the possible sets $\{n_k\}$ of configurations of m's compatible with it.

ii) Given such a configuration, $\{n_k\}$, find the degeneracy $R(\{n_k\})$ associated the possible orderings.

If we are given N punctures and two assignments of labels (j_i, m_i) and (j'_i, m'_i). Are they physically distinguishable? or are there some 'permutations' of the labels that give indistinguishable states?

That is, what is the statistics of the punctures?

As usual, we should let the theory tell us. One does not postulate any statistics. If one treats in a careful way the action of the diffeomorphisms on the punctures one learns that when one has a pair of punctures with the same labels j and m, then the punctures are indistinguishable and one should not count them twice. In all other cases the states are distinguishable.

THE COUNTING

We start with an isolated horizon, with an area a_0 and ask how many states are there compatible with the two conditions. Two relevant quantum numbers (j_I, m_I) for the Hilbert space.
Exact counting using number theory. Thus, given $(n_{1/2}, n_1, n_{3/2}, \ldots, n_{k/2})$, where $n_{s/2}$ is the number of punctures with label s we count the number of states:

$$\mathcal{N} = \sum_{\{n_s\}} \left(\frac{N!}{\Pi_s\,(n_s!)} \right) \; \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\theta \prod_s \cos(s\theta)^{n_s} \tag{3}$$

Taking the *large area approximation* $A \gg \ell_{\mathrm{Pl}}$**, and using the Stirling approximation. One gets:**

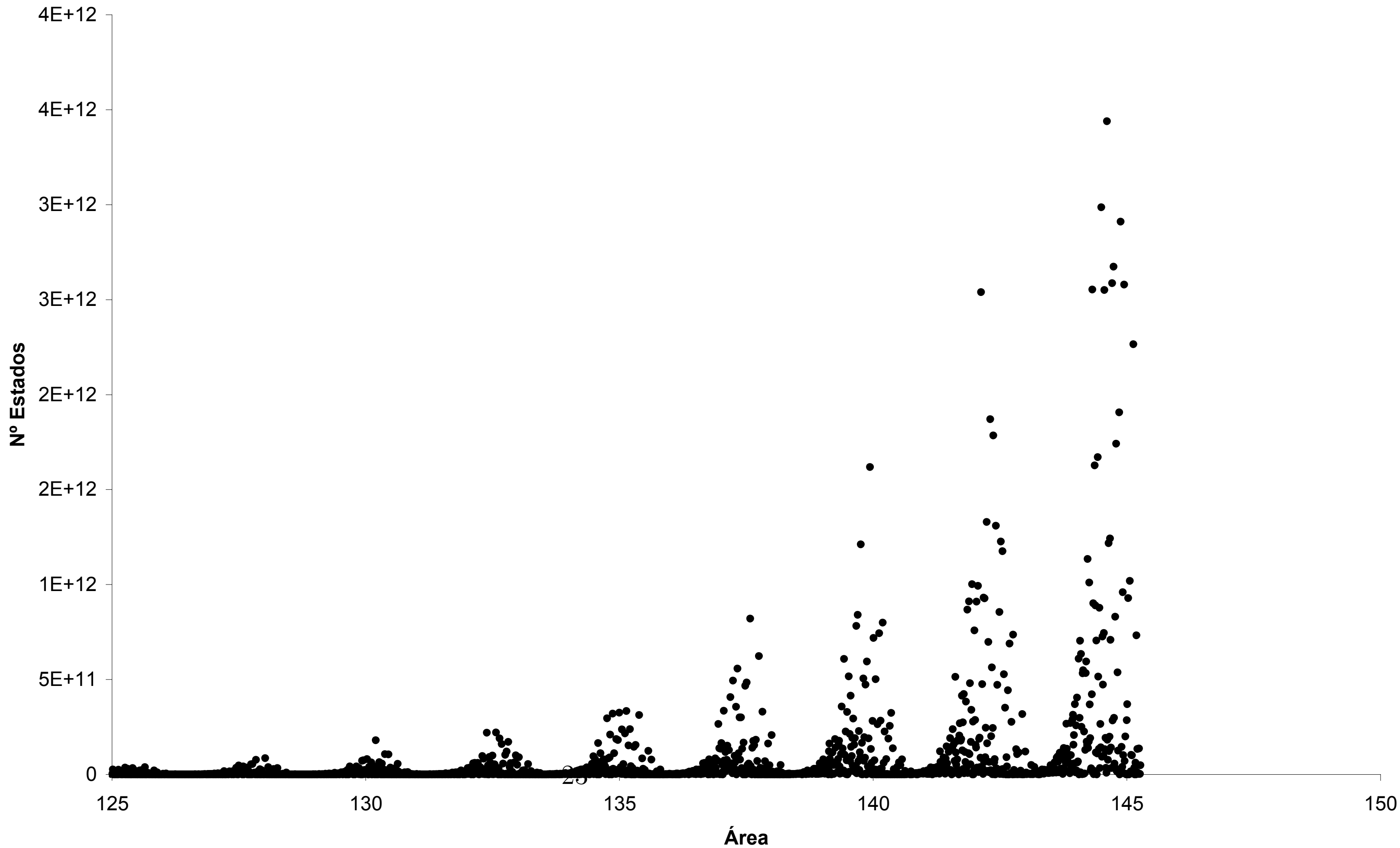
$$S = \frac{A}{4\ell_{\mathrm{Pl}}^2}\,\frac{\gamma_0}{\gamma} \tag{4}$$

with γ_0 the solution to $\sum_j 2\, e^{2\pi\,\gamma_0 \sqrt{j_i(j_i+1)}} = 1$.

The first correction to the entropy area relation is

$$S = \frac{A}{4\ell_{\rm Pl}^2}\,\frac{\gamma_0}{\gamma} - \frac{1}{2}\ln(A) + \ldots$$

- **If we want to make contact with the Bekenstein-Hawking we have to chose $\gamma = \gamma_0$.**
- **The coefficient of the logarithmic term is universal.**
- **The formalism can be generalized to more general situations, and the result is the same:**
  - **Maxwell, Dilatonic and Yang Mills Couplings (Ashtekar, Baez, AC, Krasnov, Krishnan, Fairhurst)**
  - **Cosmological, Distortion and Rotation (Ashtekar, Engle, Van der Broeck)**
  - **Non-minimal Couplings (Ashtekar, AC)**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CONCLUSIONS AND TAKE HOME MESSAGE

- **Isolated Horizons provide a consistent framework to incorporate black holes locally in equilibrium.**
- **One can consistently quantize the theory.**
- **Entropy is** *finite* **and dominant term linear in Area.**
- **Any black hole of astrophysical interest is included**
- **Analysis of Planck scale BH's shows 'quantization of entropy'.**
- **Contact with Bekenstein's heuristic model, and Mukhanov-Bekenstein in a subtle manner**

OUTLOOK

- **We have not dealt with the singularity**
- **Ashtekar-Bojowald 'paradigm' for an extended quantum space-time**
- **Based on expectations about singularity resolution coming from LQC**
- **Hawking radiation?**
- **Lost Information Puzzle**
- **Full theory: How to specify quantum black holes from the full theory?**

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HAPPY 60^{th} **BIRTHDAY ABHAY!**

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BIBLIOGRAPHY

More details can be found in:

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and

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